

~Solve by Completing the Square Notes~

MCC9-12.A.REI.4b: I can solve by completing the square.

- ❖ Certain quadratic equations can be factored into **Perfect Squares**. Factor the following quadratic expressions to see why these are called **Perfect Square Trinomials**:

• $x^2 + 6x + 9$

$$\begin{array}{r} 9 \\ 49 \\ \hline 3, 3 \end{array}$$

$$(x+3)(x+3) \rightarrow (x+3)^2$$

• $x^2 - 10x + 25$

$$\begin{array}{r} 25 \\ 1, 25 \\ \hline 5, 5 \end{array}$$

$$(x-5)(x-5) \rightarrow (x-5)^2$$

• $x^2 + 12x + 36$

$$\begin{array}{r} 36 \\ 1, 36 \\ 2, 18 \\ 3, 12 \\ \hline 4, 9 \end{array}$$

$$(x+6)(x+6) \rightarrow (x+6)^2$$

❖ **Creating a Perfect Square Trinomial**

- In the following perfect square trinomial, the constant term is missing.

$$x^2 + 14x + \underline{\quad}$$

- Find the constant term by squaring half the coefficient of the linear term. Put this number in the blank - we say that this number "**completes the square.**"

$$(14/2)^2 = 7^2$$

$$x^2 + 14x + \underline{49}$$

- ❖ **Create perfect square trinomials by finding the number that completes the square. Then factor the perfect square trinomials:**

○ $x^2 + 20x + \underline{100}$ $\left(\frac{20}{2}\right)^2 = 100$

○ $x^2 - 4x + \underline{4}$ $\left(\frac{-4}{2}\right)^2 =$

○ $x^2 + 5x + \underline{25/4}$ $\left(\frac{5}{2}\right)^2 =$

	Example 1	Example 2
the following equation by completing the square:	$x^2 + 8x - 20 = 0$	$x^2 - 6x - 35 = 0$
: Move the Constant term to left side of equation.	$x^2 + 8x = 20$	$x^2 - 6x = 35$
: Find the Number that completes the square on the left side of the equation. Take $(b/2)^2$. Add that number to both sides	$x^2 + 8x + \underline{16} = 20 + \underline{16}$ $\left(\frac{8}{2}\right)^2 = 16$ $(8/2) = 4$	$x^2 - 6x + \underline{9} = 35 + \underline{9}$ $\left(\frac{-6}{2}\right)^2 = 9$ $\left(\frac{-6}{2}\right) = 3$
: Factor the perfect square trinomial on the left side of the equation. Simplify the right side of the equation. in the form $(x \pm \frac{b}{2})^2 = \#$ Only choose 1! You must decide which one is appropriate!!	$(x + 4)^2 = 36$	$(x - 3)^2 = 44$
: Take the square root of both sides. DON'T FORGET \pm	$\sqrt{(x + 4)^2} = \sqrt{36}$ $\&$	$\sqrt{(x - 3)^2} = \sqrt{44}$
: Set up the TWO possibilities and	$x = -4 \pm 6$ $x = -4 + 6 = 2 \quad x = -4 - 6 = -10$	$x = 3 \pm 2\sqrt{11}$

Solving Quadratic Equations by Completing the Square

MCC9-12.A.REI.4b

Solving Quadratic Equations by Completing the Square

1. Rewrite so all terms containing x are on one side.
2. Find the number that completes the square on the left side of the equation. Add that number to both sides,
3. Factor the perfect square trinomial on the left side of the equation. Simplify the right side of the equation.
4. Take the square root of each side.
5. Solve for x .
6. Check your answers!!!

Examples: Solve each quadratic equation by completing the square.

$1) x^2 - 10x - 54 = 0$ $x^2 - 10x + 25 = 54 + 25$ $\sqrt{(x-5)^2} = \sqrt{79}$ $x = 5 \pm \sqrt{79}$	$2) x^2 - 18x + 77 = 0$ $x^2 - 18x + 81 = -77 + 81$ $\sqrt{(x-9)^2} = \sqrt{4}$ $x = 9 \pm 2$ $x = 11, 7$	$3) x^2 + 20x - 73 = 0$ $x^2 + 20x + 100 = 73 + 100$ $\sqrt{(x+10)^2} = \sqrt{173}$ $x = -10 \pm \sqrt{173}$
$4) x^2 + 6x - 72 = -8$ $x^2 + 6x + 9 = 64 + 9$ $\sqrt{(x+3)^2} = \sqrt{73}$ $x = -3 \pm \sqrt{73}$	$5) x^2 - 10x - 56 = 6$ $x^2 - 10x + 25 = 62 + 25$ $\sqrt{(x-5)^2} = \sqrt{87}$ $x = 5 \pm \sqrt{87}$	$6) x^2 - 14x - 75 = 8$ $x^2 - 14x + 49 = 83 + 49$ $\sqrt{(x-7)^2} = \sqrt{132}$ $x = 7 \pm 2\sqrt{33}$
$7) \frac{2x^2 - 28x + 8}{2} = 0$ $x^2 - 14x + 4 = 0$ $x^2 - 14x + 49 = -4 + 49$ $\sqrt{(x-7)^2} = \sqrt{45}$ $x = 7 \pm 3\sqrt{5}$	$8) \frac{3x^2 + 12x - 6}{3} = 0$ $x^2 + 4x - 2 = 0$ $x^2 + 4x + 4 = 2 + 4$ $\sqrt{(x+2)^2} = \sqrt{6}$ $x = -2 \pm \sqrt{6}$	$9) 5x^2 - 20x - 18 = 7$ $\frac{5x^2 - 20x}{5} = \frac{25}{5}$ $x^2 - 4x + 4 = 5 + 4$ $\sqrt{(x-2)^2} = \sqrt{9}$ $x = 2 \pm 3$ $x = 5, -1$