Complex Numbers

Introduction to Complex Numbers

* The **imaginary number**, i, is defined as the number whose square is -1.

$$i^2 = -1$$
$$i = \sqrt{-1}$$

* Imaginary and Real numbers together make up the set of complex numbers.
* A **complex number** is any number of the form $a+bi$ (or $bi+a$), where a and b are real numbers.

$$a + bi$$

real part imaginary part

Simplifying Radicals Involving Complex #s

Product Property of Square Roots	Imaginary Roots
$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$	For any non-negative real number, $\sqrt{-x} = i\sqrt{x}$

For #s 1–8, simplify the radical.

1) $\sqrt{-2}$
$i\sqrt{2}$

2) $2\sqrt{-8}$
$2i\sqrt{4}\sqrt{2}$
$4i\sqrt{2}$

3) $2\sqrt{-48}$
$2i\sqrt{16}\sqrt{3}$
$8i\sqrt{3}$

4) $4\sqrt{-50}$
$4i\sqrt{25}\sqrt{2}$
$20i\sqrt{2}$

5) $\sqrt{-98}$
$i\sqrt{49}\sqrt{2}$
$7i\sqrt{2}$

6) $-3\sqrt{-24}$
$-3i\sqrt{4}\sqrt{6}$
$-6i\sqrt{6}$

7) $\sqrt{-9}$
$3i$

8) $\sqrt{-32}$
$i\sqrt{16}\sqrt{2}$
$4i\sqrt{2}$

Powers of i

By hand, Divide the EXPONENT OF i^n by 4. The result is:			If divided in the calculator, your decimal value is
$i^1 = i$	If the remainder is ...	1	.25
$i^2 = -1$	If the remainder is ...	2	.5
$i^3 = -i$	If the remainder is ...	3	.75
$i^4 = +1$	If the remainder is ...	0	No decimal

For #s 9-12, Simplify.

9) $i^{26} = -1$
$4\overline{)26}$ = 6
24
r = 2

10) $i^{44} = 1$
$4\overline{)44}$ = 11
−44
0

11) $i^{29} = i$
$4\overline{)29}$ = 7 r 1
28
1

12) $i^{79} = -i$
$4\overline{)79}$ = 16 r 3
4
39
36

Operations with Complex Numbers

Adding Complex Numbers

- Add real parts together
- Add imaginary parts together

Example

$(-3+4i)+(-3+i)$

$-6+5i$

Subtracting Complex Numbers

- Subtract real parts
- Subtract imaginary parts

Example

$(4-5i)-(5+8i)$

$-1-13i$

Multiplying Complex Numbers

$i^2=-1$

Use box method

Example

$(-2+4i)(-1+3i)$

	-2	$4i$
-1	2	$-4i$
$3i$	$-6i$	$12i^2$

$2+-4i+-6i+12i^2$

$2-4i-6i-12 = -10i-10$

$i^2=-1$

or $-10-10i$

Dividing Complex Numbers

multiply numerator & denominator by complex conjugate of denominator

$a+bi \rightarrow a-bi$

Example

$(-3+i)\div(4-3i)$

$\frac{(-3+i)(4+3i)}{(4-3i)(4+3i)} =$

	-3	i
4	-12	$4i$
$3i$	$-9i$	$3i^2$

$\frac{-12-5i-3}{16+3} = \frac{-15-5i}{9}$

	4	$-3i$
4	16	$-12i$
$3i$	$12i$	$-3i^2$