

# Complex Numbers

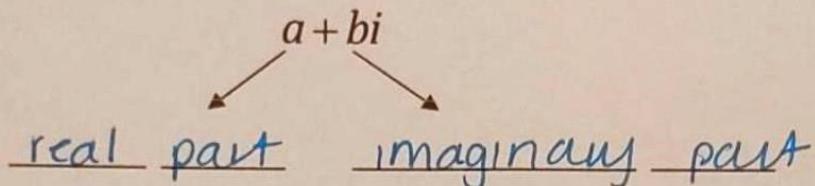
## Introduction to Complex Numbers

- \* The imaginary number,  $i$ , is defined as the number whose square is  $-1$ .

$$\boxed{i^2 = -1}$$

$$i = \sqrt{-1}$$

- \* Imaginary and real numbers together make up the set of complex numbers.
- \* A complex number is any number of the form  $a+bi$  (or  $b+ai$ ), where  $a$  and  $b$  are real numbers.



## Simplifying Radicals Involving Complex #s

Product Property of Square Roots	Imaginary Roots
$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$	For any non-negative real number, $\sqrt{-x} = i\sqrt{x}$

For #s 1-8, simplify the radical.

1)  $\sqrt{-2}$

$i\sqrt{2}$

2)  $2\sqrt{-8}$

$2i\sqrt{4}\sqrt{2}$ 
  
 $4i\sqrt{2}$

3)  $2\sqrt{-48}$

$2i\sqrt{16}\sqrt{3}$ 
  
 $8i\sqrt{3}$

4)  $4\sqrt{-50}$

$4i\sqrt{25}\sqrt{2}$ 
  
 $20i\sqrt{2}$

5)  $\sqrt{-98}$

$i\sqrt{49}\sqrt{2}$ 
  
 $7i\sqrt{2}$

6)  $-3\sqrt{-24}$

$-3i\sqrt{4}\sqrt{6}$ 
  
 $-6i\sqrt{6}$

7)  $\sqrt{-9}$

$3i$

8)  $\sqrt{-32}$

$i\sqrt{16}\sqrt{2}$ 
  
 $4i\sqrt{2}$

## Powers of $i$

By hand, Divide the EXPONENT OF $i^n$ by 4. The result is:			If divided in the calculator, your decimal value is
$i^1 = i$	If the remainder is ...	1	.25
$i^2 = -1$	If the remainder is ...	2	.5
$i^3 = -i$	If the remainder is ...	3	.75
$i^4 = +1$	If the remainder is ...	0	No decimal

For #s 9-12, Simplify.

9)  $i^{26} = -1$

$$4 \overline{) 26} \\ \underline{-24} \\ 2 \\ r=2$$

10)  $i^{44} = 1$

$$4 \overline{) 44} \\ \underline{-44} \\ 0$$

11)  $i^{29} = i$

$$4 \overline{) 29} \\ \underline{-28} \\ 1$$

12)  $i^{79} = -i$

$$4 \overline{) 79} \\ \underline{-4} \\ 39 \\ \underline{-36} \\ 3$$

# Operations with Complex Numbers

Adding Complex Numbers

- Add real parts together
- Add imaginary parts together

$$(-3 + 4i) + (-3 + i)$$

$$-6 + 5i$$

Example

Subtracting Complex Numbers

- Subtract real parts
- Subtract imaginary parts

$$(4 - 5i) - (5 + 8i)$$

$$-1 - 13i$$

Example

Multiplying Complex Numbers

$$i^2 = -1$$

Use box method

$$(-2 + 4i)(-1 + 3i)$$

Example

-2	4i	
-1	2	-4i
3i	-6i	12i^2

$$\begin{array}{r} -4i + -4i + 12i^2 \\ \hline 2 -4i -6i -12 \end{array} = -10i - 10 \quad i^2 = -1$$

Dividing Complex Numbers

multiply numerator & denominator by complex conjugate  
of denominator  
 $a+bi \rightarrow a-bi$

$$(-3+i) \div (4-3i) \quad \text{or} \quad -10-10$$

Example

$$\frac{(-3+i)(4+3i)}{(4-3i)(4+3i)} =$$

-3	1	
4	-12	4i
3i	-9i	3i^2

$$\frac{-12-5i-3-15-5i}{16+3} = \frac{-15-10i}{9}$$

4	-3i	
4	16	-12i
3i	12i	-3i^2