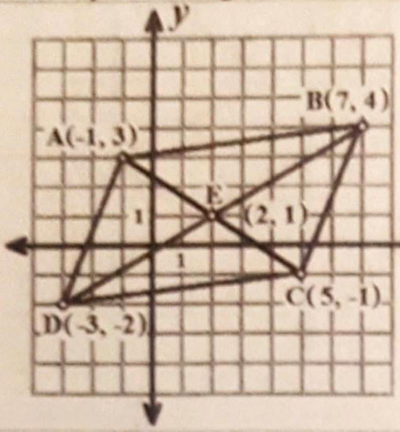
PROOFS USING DISTANCE AND SLOPE

Distance	Midpoint	Slope
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\sqrt{(\Delta x)^2+(\Delta y)^2}$	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$	$\frac{y_2-y_1}{x_2-x_1} = \frac{\Delta y}{\Delta x}$

The diagonals of a parallelogram bisect each other. For #1-2, prove that ABCD is a parallelogram for each case by showing that AE is congruent to EC and BE is congruent to ED.

① 

Midpoint of AC:

$\left(\frac{-1+5}{2}, \frac{3+-1}{2}\right)$

$\left(\frac{4}{2}, \frac{2}{2}\right)$

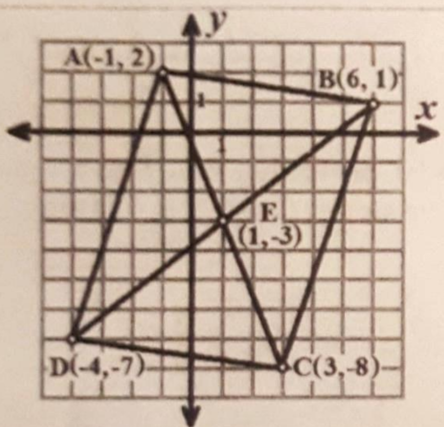
$(2, 1)$

Midpoint of DB:

$\left(\frac{-3+7}{2}, \frac{-2+4}{2}\right)$

$\left(\frac{4}{2}, \frac{2}{2}\right)$

$(2, 1)$

② 

Midpoint of AC:

$\left(\frac{-1+3}{2}, \frac{2+-8}{2}\right)$

$\left(\frac{2}{2}, \frac{-6}{2}\right)$

$(1, -3)$

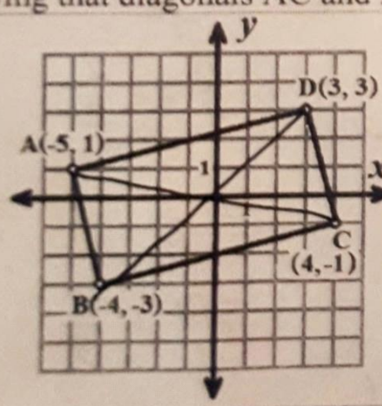
Midpoint of DB:

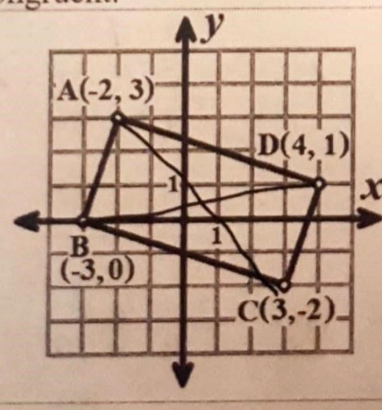
$\left(\frac{-4+6}{2}, \frac{-7+1}{2}\right)$

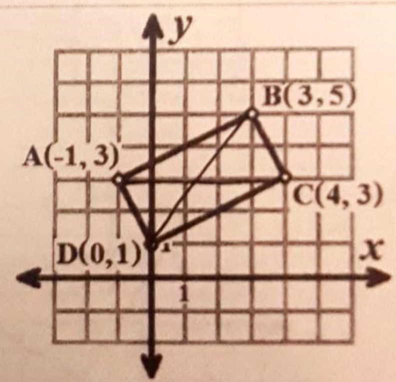
$\left(\frac{2}{2}, \frac{-6}{2}\right)$

$(1, -3)$

The diagonals of a rectangle are congruent. For #3-5, prove that ABCD is a rectangle for each case by showing that diagonals AC and BD are congruent.

③ 

④ 

⑤ 

③

$AC = \sqrt{(4 - -5)^2 + (-1 - 1)^2}$
$= \sqrt{(9)^2 + (-2)^2}$
$= \sqrt{(81) + (4)} = \sqrt{85}$

$BD = \sqrt{(3 - -3)^2 + (3 - -4)^2}$
$= \sqrt{(6)^2 + (7)^2}$
$= \sqrt{(36) + (49)} = \sqrt{85}$

④

$AC = \sqrt{(3 - -2)^2 + (-2 - 3)^2}$
$= \sqrt{(5)^2 + (-5)^2}$
$= \sqrt{(25) + (25)} = \sqrt{50} = 5\sqrt{2}$

$BD = \sqrt{(4 - -3)^2 + (1 - 0)^2}$
$= \sqrt{(7)^2 + (1)^2}$
$= \sqrt{(49) + (1)} = \sqrt{50} = 5\sqrt{2}$

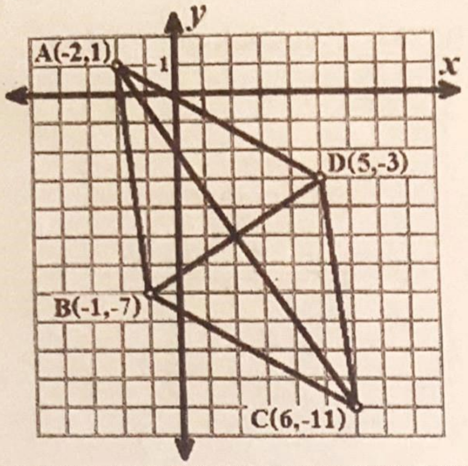
⑤

$AC = \sqrt{(4 - -1)^2 + (3 - 3)^2}$
$= \sqrt{(5)^2 + (0)^2}$
$= \sqrt{(25) + (0)} = \sqrt{25} = 5$

$BD = \sqrt{(0 - 3)^2 + (1 - 5)^2}$
$= \sqrt{(-3)^2 + (-4)^2}$
$= \sqrt{(9) + (16)} = \sqrt{25} = 5$

The diagonals of a rhombus are perpendicular. For #6-8, prove that ABCD is a rhombus for each case by showing that the slopes of diagonals AC and BD are opposite reciprocals of each other.

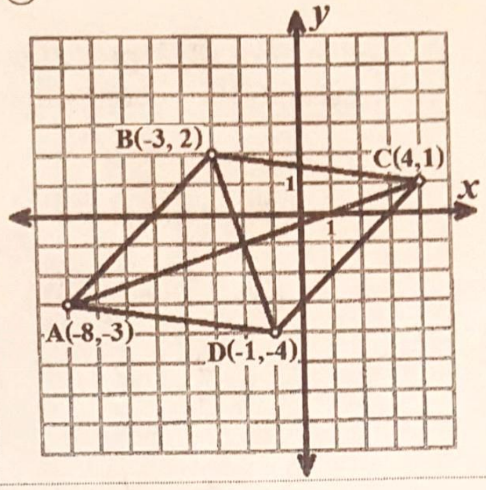
⑥



$m_{AC} = \dfrac{-11 - 1}{6 - {-2}} = \dfrac{-12}{8} = \dfrac{-3}{2}$

$m_{BD} = \dfrac{-3 - -7}{5 - -1} = \dfrac{4}{6} = \dfrac{2}{3}$

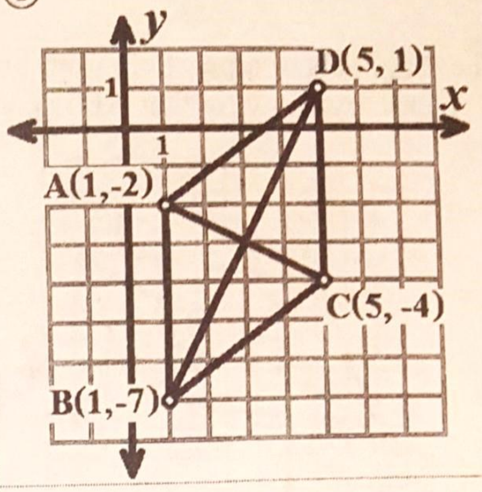
⑦



$m_{AC} = \dfrac{1 - -3}{4 - -8} = \dfrac{4}{12} = \dfrac{1}{3}$

$m_{BD} = \dfrac{-4 - 2}{-1 - -3} = \dfrac{-6}{2} = -3$

⑧

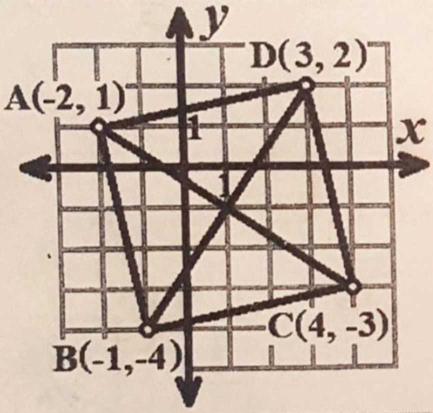


$m_{AC} = \dfrac{-4 - -2}{5 - 1} = \dfrac{-2}{4} = \dfrac{-1}{2}$

$m_{BD} = \dfrac{1 - -7}{5 - 1} = \dfrac{8}{4} = 2$

The diagonals of a square are both congruent and perpendicular. For #9-10, prove that ABCD is a square for each case by showing that diagonal AC is congruent to BD and that the slopes of AC and BD are opposite reciprocals. Show work.

⑨



AC

$= \sqrt{(4 - -2)^2 + (-3 - 1)^2}$

$= \sqrt{(6)^2 + (-4)^2}$

$= \sqrt{(36) + (16)} = \sqrt{52} = 2\sqrt{13}$

BD

$= \sqrt{(3 - -1)^2 + (2 - -4)^2}$

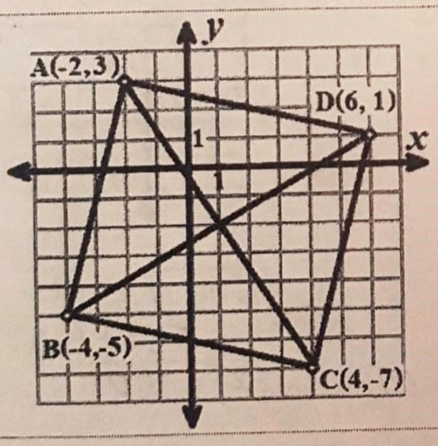
$= \sqrt{(4)^2 + (6)^2}$

$= \sqrt{(16) + (36)} = \sqrt{52} = 2\sqrt{13}$

$m_{AC} = \dfrac{-3 - 1}{4 - -2} = \dfrac{-4}{6} = \dfrac{-2}{3}$

$m_{BD} = \dfrac{2 - -4}{3 - -1} = \dfrac{6}{4} = \dfrac{3}{2}$

⑩



AC

$= \sqrt{(4 - -2)^2 + (-7 - 3)^2}$

$= \sqrt{(6)^2 + (-10)^2}$

$= \sqrt{(36) + (100)} = \sqrt{136}$

$2\sqrt{34}$

BD

$= \sqrt{(6 - -4)^2 + (1 - -5)^2}$

$= \sqrt{(10)^2 + (6)^2}$

$= \sqrt{(100) + (36)} = \sqrt{136}$

$2\sqrt{34}$

$m_{AC} = \dfrac{-7 - 3}{4 - -2} = \dfrac{-10}{6} = \dfrac{-5}{3}$

$m_{BD} = \dfrac{1 - -5}{6 - -4} = \dfrac{6}{10} = \dfrac{3}{5}$