

Evaluating Trigonometric Functions
Precalculus

Name: Key
Date: _____

Use the given values to evaluate the remaining trigonometric functions.

1) $\sin \theta = \frac{2}{5}$ $0^\circ < \theta < 90^\circ$ Q1

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + 2^2 &= 5^2 \\x^2 + 4 &= 25 \\x^2 &= 21 \\x &= \sqrt{21}\end{aligned}$$

$x = \sqrt{21}$ $y = 2$ $r = 5$

+ $\sin \theta = \frac{2}{5}$ $\csc \theta = \frac{5}{2}$

+ $\cos \theta = \frac{\sqrt{21}}{5}$ $\sec \theta = \frac{5\sqrt{21}}{21}$

+ $\tan \theta = \frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$ $\cot \theta = \frac{\sqrt{21}}{2}$

3) $\sin \theta = \frac{1}{4}$ $0^\circ < \theta < \frac{\pi}{2}$ Q1

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + 1^2 &= 4^2 \\x^2 + 1 &= 16 \\x^2 &= 15 \\x &= \sqrt{15}\end{aligned}$$

$x = \sqrt{15}$ $y = 1$ $r = 4$

$\sin \theta = \frac{1}{4}$ $\csc \theta = \frac{4}{1}$

$\cos \theta = \frac{\sqrt{15}}{4}$ $\sec \theta = \frac{4\sqrt{15}}{15}$

$\tan \theta = \frac{1}{\frac{\sqrt{15}}{4}} = \frac{4}{\sqrt{15}}$ $\cot \theta = \frac{\sqrt{15}}{4}$

5) $\csc \theta = \frac{\sqrt{11}}{3}$ $\frac{\pi}{2} < \theta < \pi$ Q2

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + 3^2 &= (\sqrt{11})^2 \\x^2 + 9 &= 11\end{aligned}$$

$x = \sqrt{2}$

$x = \sqrt{2}$ $y = 3$ $r = \sqrt{11}$

+ $\sin \theta = \frac{3}{\sqrt{11}}$ $\csc \theta = \frac{\sqrt{11}}{3}$

- $\cos \theta = \frac{\sqrt{2}}{\sqrt{11}} = \frac{\sqrt{22}}{11}$ $\sec \theta = \frac{\sqrt{22}}{2}$

- $\tan \theta = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$ $\cot \theta = \frac{\sqrt{2}}{3}$

2) $\tan \theta = \frac{\sqrt{3}}{4}$ $0 < \theta < \frac{\pi}{2}$ Q1

$$\begin{aligned}x^2 + y^2 &= r^2 \\4^2 + (\sqrt{3})^2 &= r^2 \\16 + 3 &= r^2 \\r &= \sqrt{19}\end{aligned}$$

$$\begin{aligned}x &= 4 \\y &= \sqrt{3} \\r &= \sqrt{19} \\+ \sin \theta &= \frac{\sqrt{3}}{\sqrt{19}} = \frac{\sqrt{57}}{19} \\+ \csc \theta &= \frac{\sqrt{19}}{\sqrt{3}} = \frac{\sqrt{57}}{3} \\+ \cos \theta &= \frac{4}{\sqrt{19}} = \frac{4\sqrt{19}}{19} \\+ \sec \theta &= \frac{\sqrt{19}}{4} \\+ \tan \theta &= \frac{\sqrt{3}}{4} \\+ \cot \theta &= \frac{4\sqrt{3}}{3}\end{aligned}$$

4) $\cos \theta = -\frac{2}{3}$ $90^\circ < \theta < 180^\circ$ Q2

$$\begin{aligned}x^2 + y^2 &= r^2 \\(-2)^2 + 4^2 &= 3^2 \\4 + 16 &= 9 \\r &= \sqrt{20} = \sqrt{4} \cdot \sqrt{5}\end{aligned}$$

$$\begin{aligned}x &= -2 \\y &= \sqrt{5} \\r &= \sqrt{20} = \sqrt{4} \cdot \sqrt{5} \\+ \sin \theta &= \frac{\sqrt{5}}{\sqrt{20}} = \frac{\sqrt{5}}{4} \\+ \csc \theta &= \frac{4\sqrt{5}}{5} \\- \cos \theta &= \frac{-2}{\sqrt{20}} = \frac{-2}{4} = \frac{-1}{2} \\- \sec \theta &= \frac{-4}{\sqrt{20}} = \frac{-4}{4} = -1 \\- \tan \theta &= \frac{\sqrt{5}}{2} \\- \cot \theta &= \frac{-2\sqrt{5}}{5}\end{aligned}$$

6) $\sec \theta = -\frac{5}{4}$ $90^\circ < \theta < 180^\circ$ Q2

$$\begin{aligned}x^2 + y^2 &= r^2 \\(-4)^2 + y^2 &= 25 \\16 + y^2 &= 25 \\y^2 &= 9 \\y &= 3\end{aligned}$$

$$\begin{aligned}x &= -4 \\y &= 3 \\r &= \sqrt{25} = 5 \\+ \sin \theta &= \frac{3}{5} \\+ \csc \theta &= \frac{5}{3} \\- \cos \theta &= \frac{-4}{5} \\- \sec \theta &= \frac{-5}{4} \\- \tan \theta &= \frac{-3}{4} \\- \cot \theta &= \frac{-4}{3}\end{aligned}$$

$$7) \sin \theta = -\frac{1}{3} \quad 180^\circ < \theta < 270^\circ \quad Q3$$

$$8) \tan \theta = \frac{2}{3} \quad \pi < \theta < \frac{3\pi}{2} \quad Q3$$

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + (-1)^2 &= 3^2 \\x^2 + 1 &= 9 \\-1 &\downarrow\end{aligned}$$

$$x = 2\sqrt{2}, y = -1, r = 3$$

$$-\sin \theta = -\frac{1}{3}, -\csc \theta = -\frac{3}{1}$$

$$-\cos \theta = -\frac{2\sqrt{2}}{3}, -\sec \theta = -\frac{3\sqrt{2}}{1}$$

$$+\tan \theta = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}, \cot \theta = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\begin{aligned}x^2 + y^2 &= r^2 \\2^2 + 3^2 &= 12 \\r &= \sqrt{13}\end{aligned}$$

$$\begin{aligned}x &= \frac{3}{\sqrt{13}}, y = \frac{2}{\sqrt{13}}, r = \sqrt{13} \\-\sin \theta &= \frac{-2}{\sqrt{13}}, -\csc \theta = \frac{-2\sqrt{13}}{13} \\-\cos \theta &= \frac{-3}{\sqrt{13}}, -\sec \theta = \frac{-3\sqrt{13}}{13} \\+\tan \theta &= \frac{2}{\sqrt{13}}, +\cot \theta = \frac{3}{\sqrt{13}}\end{aligned}$$

$$9) \sec \theta = -\frac{7}{5} \quad 180^\circ < \theta < 270^\circ \quad Q3$$

$$10) \cos \theta = \frac{1}{8} \quad \pi < \theta < \frac{3\pi}{2} \quad Q3$$

$$\begin{aligned}x^2 + y^2 &= r^2 \\(-5)^2 + 1^2 &= 7^2 \\5^2 + 1^2 &= 24\end{aligned}$$

$$\begin{aligned}x^2 + y^2 &= r^2 \\1^2 + 0^2 &= 8^2 \\r &= 3\sqrt{7}\end{aligned}$$

$$\begin{aligned}x &= -5, y = 2\sqrt{2}, r = 7 \\-\sin \theta &= \frac{2\sqrt{4}}{7}, -\csc \theta = \frac{7\sqrt{4}}{12} \\-\cos \theta &= \frac{5}{7}, -\sec \theta = \frac{7}{15}\end{aligned}$$

$$\begin{aligned}x &= 1, y = 3\sqrt{7}, r = 8 \\-\sin \theta &= \frac{-3\sqrt{7}}{8}, -\csc \theta = \frac{-8\sqrt{7}}{21} \\+\cos \theta &= \frac{1}{8}, +\sec \theta = \frac{8}{1}\end{aligned}$$

$$+ \tan \theta = \frac{2\sqrt{6}}{5}, + \cot \theta = \frac{5\sqrt{6}}{12}$$

$$-\tan \theta = \frac{3\sqrt{7}}{1}, -\cot \theta = \frac{-\sqrt{7}}{21}$$

$$11) \cot \theta = -\frac{4}{3} \quad 270^\circ < \theta < 360^\circ \quad Q4$$

$$12) \cot \theta = -8 \quad \pi < \theta < \frac{3\pi}{2} \quad Q4$$

$$\begin{aligned}x^2 + y^2 &= r^2 \\(-4)^2 + 3^2 &= 7^2 \\16 + 9 &= 49 \\r &= 5\end{aligned}$$

$$\begin{aligned}x^2 + y^2 &= r^2 \\(-8)^2 + 1^2 &= 1^2 \\r &= 1\end{aligned}$$

$$\begin{aligned}x &= -4, y = 3, r = 5 \\-\sin \theta &= \frac{3}{5}, -\csc \theta = \frac{-5}{3} \\+\cos \theta &= \frac{4}{5}, +\sec \theta = \frac{5}{4} \\-\tan \theta &= \frac{3}{4}, -\cot \theta = \frac{-4}{3}\end{aligned}$$

$$\begin{aligned}x &= -8, y = 1, r = \sqrt{65} \\-\sin \theta &= \frac{-1}{\sqrt{65}}, -\csc \theta = \frac{-\sqrt{65}}{1} \\+\cos \theta &= \frac{-8}{\sqrt{65}}, +\sec \theta = \frac{\sqrt{65}}{-8} \\-\tan \theta &= \frac{1}{8}, -\cot \theta = \frac{-8}{1}\end{aligned}$$