Proving Quadrilaterals

Distance d = $\sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$

Midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Slope = m = $\frac{y_1-y_2}{x_1-x_2}$

Show the diagonals of the parallelogram bisect each other.

1. Prove that the quadrilateral PQRS with the coordinates P(0, 2), Q(4, 8), R(7, 6) and S(3, 0) is a parallelogram.

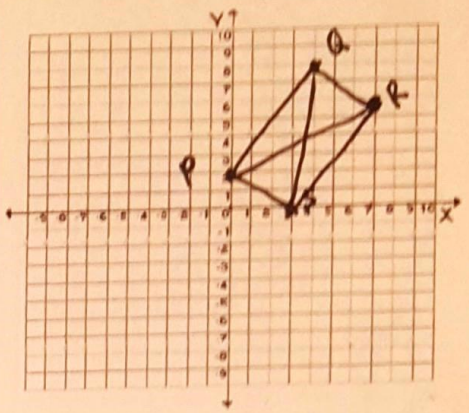
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

P(0,2) R(7,6) Q(4,8), S(3,0)

Midpoint of PR: $\left(\frac{0+7}{2}, \frac{6+2}{2}\right)$ Midpoint of QS: $\left(\frac{3+4}{2}, \frac{0+8}{2}\right)$

(3.5, 4) (3.5, 4)

Since midpoints are =, then diagonals bisect each other ✓



2. Prove that the quadrilateral LMNO with the coordinates L(-2,3), M(4,3), N(2,-2) and O(-4,-2) is a parallelogram.

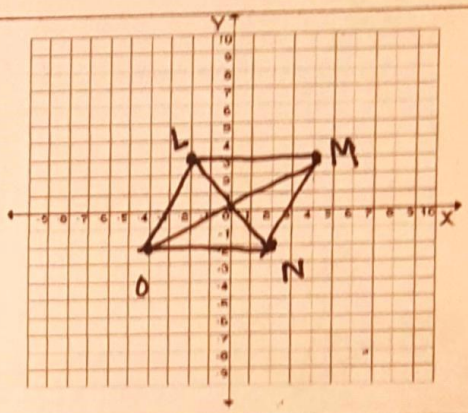
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

L(-2,3) N(2,-2) M(4,3) O(-4,-2)

Midpoint of LN: $\left(\frac{-2+2}{2}, \frac{3+-2}{2}\right)$ Midpoint of MO: $\left(\frac{4+-4}{2}, \frac{3+2}{2}\right)$

(0, .5) (0, .5)

Since midpoints are =, then diagonals bisect each other.



Show the diagonals of the rectangle are congruent

3. Prove a quadrilateral GHIJ with vertices G(0,5), H(6,9), I(8,6) and J(2,2) is a rectangle.

: $d = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$

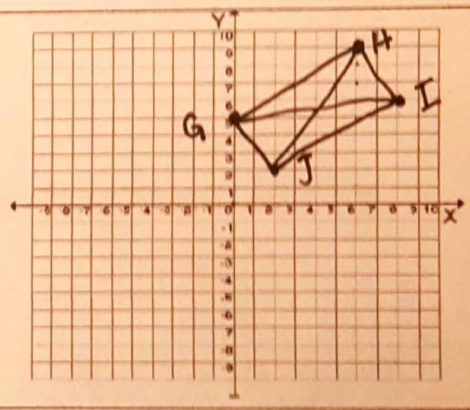
G(0,5), I(8,6) H(6,9), (2,2)

GI: $= \sqrt{(8-0)^2 + (6-5)^2}$ HJ: $= \sqrt{(2-6)^2 + (2-9)^2}$

$= \sqrt{(8)^2 + (1)^2}$ $= \sqrt{(-4)^2 + (-7)^2}$

$= \sqrt{(64) + (1)} = \sqrt{65}$ $= \sqrt{(16) + (49)} = \sqrt{65}$

Since distance is =, the diagonals are ≅.



4. The vertices of quadrilateral COAT are C(0,0), O(5,0), A(5,2) and T(0,2). Prove that COAT is a rectangle.

: $d = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$

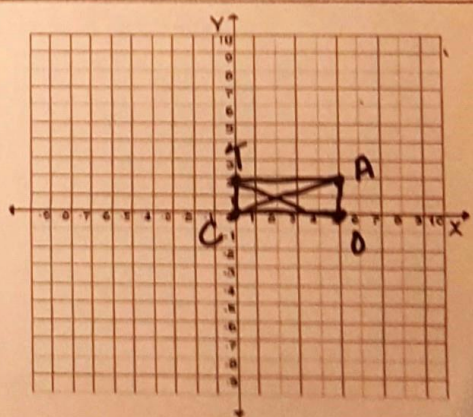
C(0,0) A(5,2) O(5,0) T(0,2)

CA: $= \sqrt{(5-0)^2 + (2-0)^2}$ OT: $= \sqrt{(0-5)^2 + (2-0)^2}$

$= \sqrt{(5)^2 + (2)^2}$ $= \sqrt{(-5)^2 + (2)^2}$

$= \sqrt{(25) + (4)} = \sqrt{29}$ $= \sqrt{(25) + (4)} = \sqrt{29}$

Since distance is =, the diagonals are ≅.



Show the diagonals of the rhombus are perpendicular.

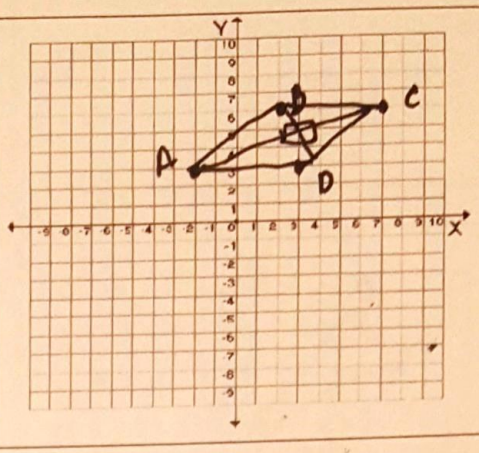
5. Prove that a quadrilateral ABCD with the vertices A(-2,3), B(2,6), C(7,6) and D(3,3) is a rhombus.

$m = \frac{y_1 - y_2}{x_1 - x_2}$

A(-2,3), C(7,6) $\qquad$ B(2,6), (3,3)

$m_{AC} = \frac{7 - -2}{6 - 3} = \frac{9}{3} = 3$ $\qquad$ $m_{BD} = \frac{3 - 2}{3 - 6} = \frac{1}{-3} = -\frac{1}{3}$

since slopes are ⊥, the diagonals are ⊥



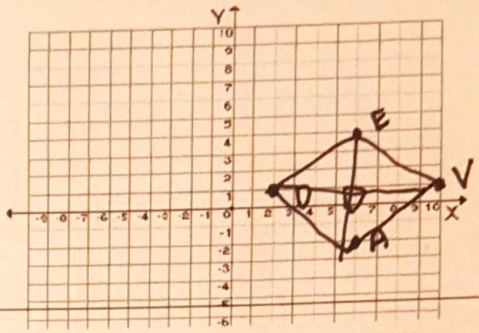
6. Prove that the quadrilateral DAVE with the vertices D(2,1), A(6,-2), V(10,1) and E(6,4) is a rhombus.

$m = \frac{y_1 - y_2}{x_1 - x_2}$

D(2,1) V:(10,1) $\qquad$ A(6,-2) E(6,4)

$m_{DV} = \frac{10 - 2}{1 - 1} = \frac{8}{0} =$ $\qquad$ $m_{AE} = \frac{6 - -2}{4 - 6} = \frac{8}{-2} = \frac{0}{6}$

since slopes are ⊥, the diagonals are ⊥



Show the diagonals of a square are both congruent and perpendicular.

7. Prove that the quadrilateral ABCD with vertices A(0,0), B(4,3), C(7,-1) and D(3,-4) is a square.

$d = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$ $\qquad$ $m = \frac{y_1 - y_2}{x_1 - x_2}$

A(0,0) C(7,-1) $\qquad$ B(4,3), D(3,-4)

AC: $= \sqrt{(7 - 0)^2 + (-1 - 0)^2}$ $\qquad$ BD: $= \sqrt{(3 - 4)^2 + (-4 - 3)^2}$

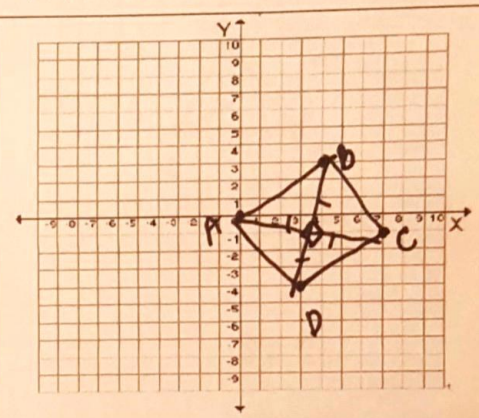
$= \sqrt{(7)^2 + (-1)^2}$ $\qquad$ $= \sqrt{(-1)^2 + (-7)^2}$

$= \sqrt{(49) + (1)} = \sqrt{50}$ $\qquad$ $= \sqrt{(1) + (49)} = \sqrt{50}$

AND

$m_{AC} = \frac{-1 - 0}{7 - 0} = \frac{-1}{7} = -\frac{1}{7}$ $\qquad$ $m_{BD} = \frac{-4 - 3}{3 - 4} = \frac{-7}{-1} = 7$

slopes are ⊥, diagonals are ⊥, & distance is =, diagonals are ≅



8. Prove that the quadrilateral ABCD with vertices A(2,2), B(5,-2), C(9,1) and D(6,5) is a square.

$d = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$ $\qquad$ $m = \frac{y_1 - y_2}{x_1 - x_2}$

A(2,2) C(9,1) $\qquad$ B(5,-2), D(6,5)

AC: $= \sqrt{(9 - 2)^2 + (1 - 2)^2}$ $\qquad$ BD: $= \sqrt{(6 - 5)^2 + (5 - -2)^2}$

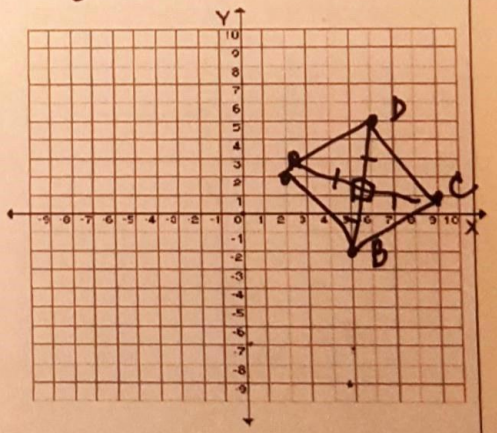
$= \sqrt{(7)^2 + (-1)^2}$ $\qquad$ $= \sqrt{(1)^2 + (7)^2}$

$= \sqrt{(49) + (1)} = \sqrt{50}$ $\qquad$ $= \sqrt{(1) + (49)} = \sqrt{50}$

AND

$m_{AC} = \frac{1 - 2}{9 - 2} = \frac{-1}{7} = -\frac{1}{7}$ $\qquad$ $m_{BD} = \frac{5 - -2}{6 - 5} = \frac{7}{1} = 7$

slopes are ⊥, diagonals are ⊥, distance is =, diagonals are ≅.



If a quadrilateral is both a rectangle and a rhombus, then it is a square.