Dilations and Scale Factor

Dilations are a resizing of the image. They change the lengths of the segments but NOT the angles. Unlike the other transformations we have learned about, dilation is not an isometry (a transformation in which the original figure and its image are congruent).

The first step to performing a dilation is to multiply by a scale factor. What is a scale factor? A **scale factor** is the number used to multiply the lengths of a figure to stretch or shrink it to a similar image.

- If a scale factor is less than 1, the resulting image will be a reduction
- If a scale factor is greater than 1, the resulting image will be an enlargement
- If a scale factor is equal to 1, the resulting image will be congruent.

TIP:

It may be helpful to convert fractions and percents to decimals to determine if the scale factor is greater than, less than, or equal to 1.

Practice: Determine if the following scale factors will result in an enlargement, reduction, or congruence:

1) $\frac{1}{4}$ reduction
2) .75 reduction
3) 125% enlargement
4) $\frac{15}{7} \approx 2.14$ enlargement
5) 100% = 1 congruence

Now that we have developed an understanding of scale factors, we can begin performing dilations.

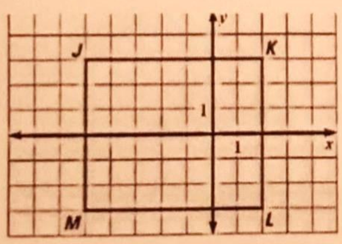
Steps for performing dilations:

1) Multiply Both coordinates by the given scale factor.
2) Simplify.
3) graph (if required).

Example:

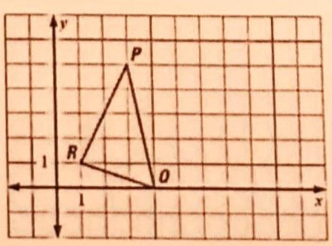
Use the given scale factor to find the coordinates of the vertices of the image of the polygon.

- $k = \frac{1}{2}$



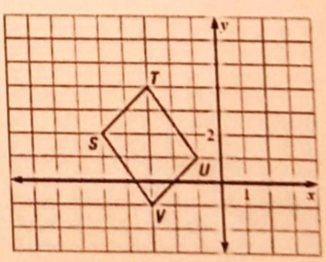
J (− 6, 3) → (−3, 1.5)
K (2, 3) → (1, 1.5)
L (2, − 3) → (1, −1.5)
M (− 6, − 3) → (−3, −1.5)

2) **k = 2**



P (3, 5) → (6, 10)
Q (4, 0) → (8, 0)
R (1, 1) → (2, 2)

3) **k = 4**



S (− 5, 2) → (−20, 8)
T (− 3, 4) → (−12, 16)
U (− 1, 1) → (−4, 4)
V (−3, − 1) → (−12, −4)

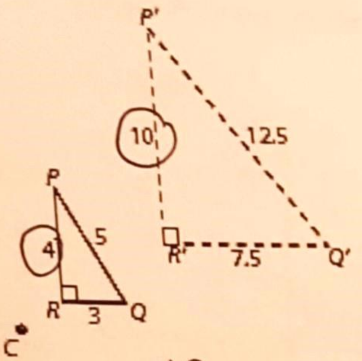
Finding the Scale Factor and Center of Dilation

Sometimes, we may be asked to work backwards. We may be given an image and pre-image and be asked to find the scale factor. How can we do this?

The scale factor is the ratio of

$\frac{\text{image distance}}{\text{pre-image distance}}$ or $\frac{\text{new image}}{\text{original image}}$

Example: Determine the scale factor and whether the dilation is an enlargement, reduction, or congruency transformation. The dotted figure is the new image.



$\frac{10}{4} = 2.5$ $\frac{12.5}{5} = 2.5$

$\frac{7.5}{3} = 2.5$

scale factor = 2.5

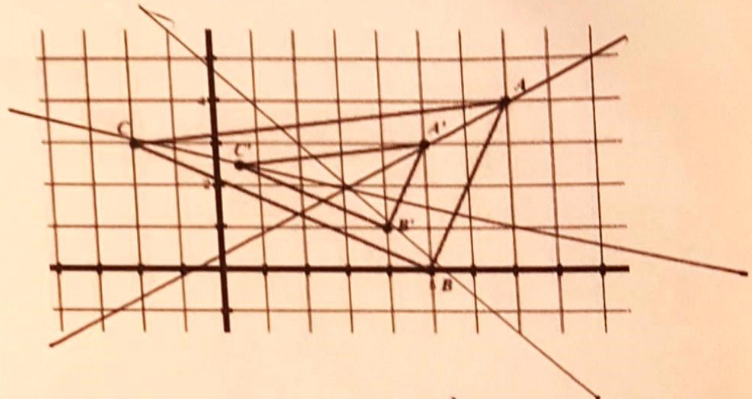
enlargement

The center of dilation is a constant point on a surface from which all other points are either enlarged or compressed.

To find the center of dilation given two images (a pre-image and image) we connect corresponding points from an image and pre-image. **The intersection of the lines is the center of dilation.**

To ensure accuracy, use the slope between corresponding points to construct the lines.

Example: Find the center of dilation.



(3,2)

Geometry
Name_______________________________

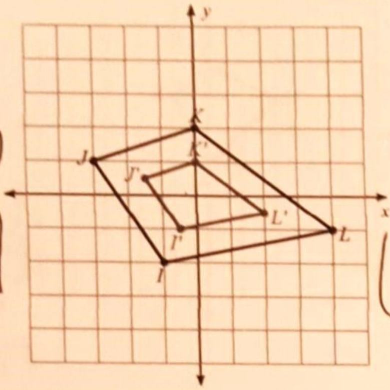
Dilations Around the Origin HW

Date________________ Period____

Write a rule to describe each transformation.

1)

J=(-3,1)
J'=(-1.5,.5)
K=(0,2)
K'=(0,1)

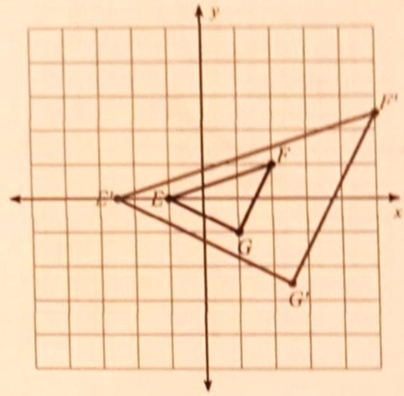


$K=\frac{1}{2}$

(Kx, Ky)

$(\frac{1}{2}x, \frac{1}{2}y)$

2)

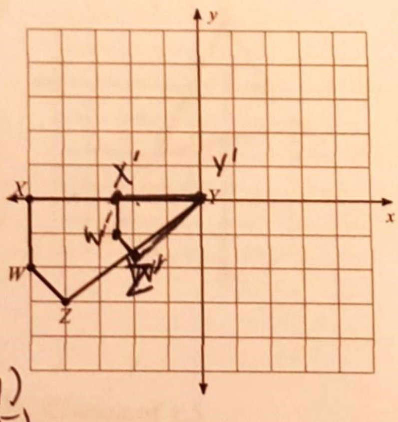


F(2,1)
F'(5,2.5)
(2.5x, 2.5y)
K=2.5)

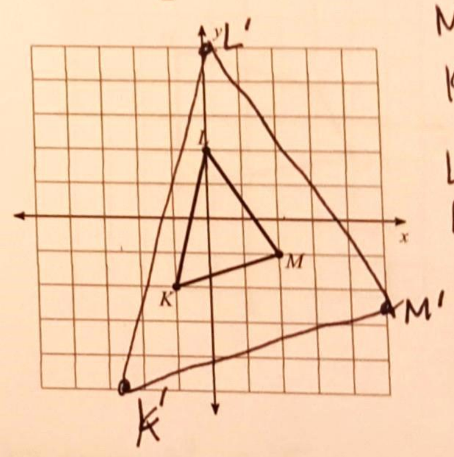
Graph the image of the figure using the transformation given.

3) dilation of $\frac{1}{2}$

X: (-5,0)
Y: (0,0)
W: (-5,-2)
Z: (-4,-3)
X': (-2.5,0)
Y': (0,0)
W': (-2.5,-1)
Z': (-2,-1.5)



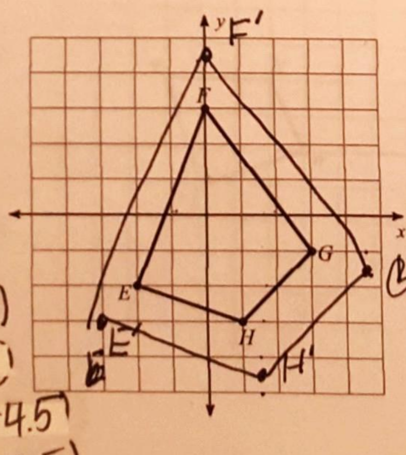
4) dilation of $\frac{5}{2}$ = 2.5



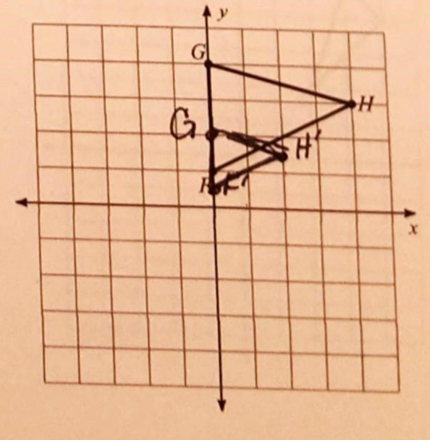
L: (0,2)
M: (2,-1)
K: (-1,-2)
L': (0,5)
M': (5,-2.5)
K': (-2.5,-5)

5) dilation of 1.5

F: (0,3)
E: (-2,-2)
H: (1,-3)
G: (3,-1)
F': (0,4.5)
E': (-3,-3)
H': (1.5,-4.5)
G': (4.5,-1.5)



6) dilation of $\frac{1}{2}$

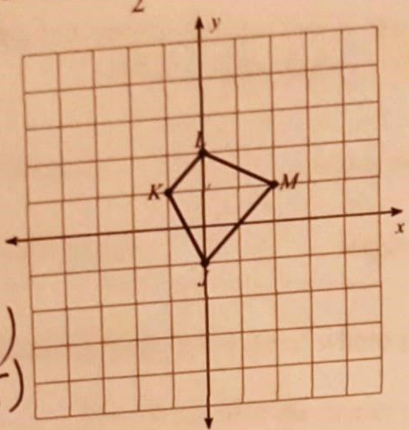


G: (0,4)
F: (0,1)
H: (4,3)

G': (0,2)
F': (0,.5)
H': (2,1.5)

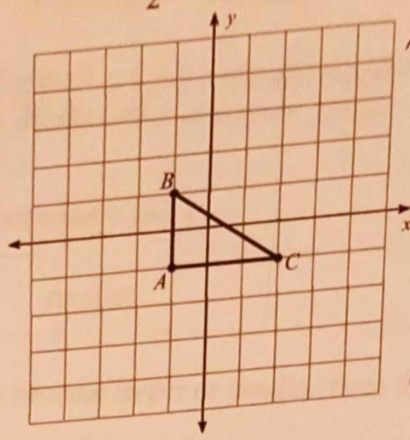
Find the coordinates of the vertices of each figure after the given transformation.

7) dilation of $\frac{3}{2}$ $= 1.5$



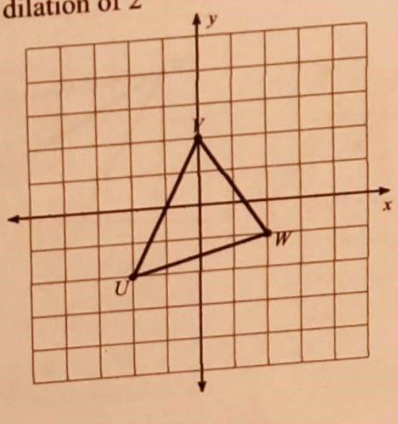
J: (0,-1)
K: (-1,1)
L: (0,2)
M: (2,1)
J': (0,-1.5)
K': (-1.5,1.5)
L': (0, 3)
M': (3,1.5)

8) dilation of $\frac{1}{2}$ $= .5$



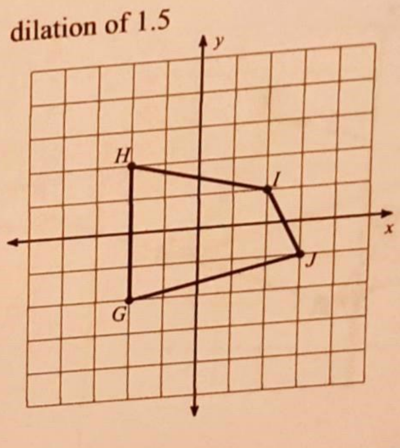
A: (-1,-1)
B: (-1,1)
C: (2,-1)
A': $(-\frac{1}{2}, -\frac{1}{2})$
B': $(-\frac{1}{2}, \frac{1}{2})$
C': $(1, -\frac{1}{2})$

9) dilation of 2



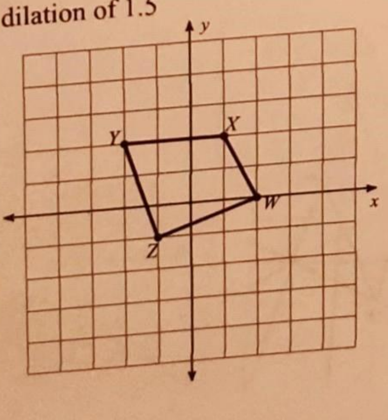
V: (0,2)
U: (-2,-2)
W: (2,-1)
V': (0,4)
U': (-4,-4)
M: (4,-2)

10) dilation of 1.5



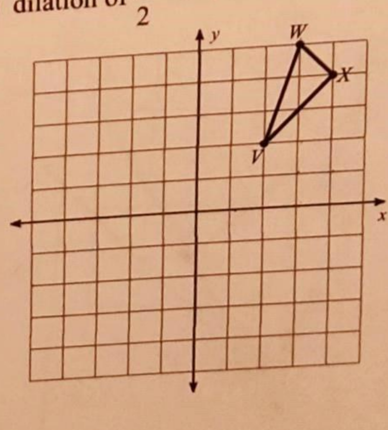
H: (-2,2)
G: (-2,-2)
I: (2,1)
J: (3,-1)
H': (-3,3)
G': (-3,-3)
I': (3,1.5)
J': (4.5,-1.5)

11) dilation of 1.5



X: (1,2)
Y: (-2,2)
W: (2,0)
Z: (-1,-1)
X': (1.5,3)
Y': (-3, 3)
W': (3,0)
Z': (-1.5,-1.5)

12) dilation of $\frac{1}{2}$ $= .5$



X: (4,4)
V: (2,2)
W: (3,5)
X': (2,2)
V': (1,1)
W': (1.5,2.5)

PROPERTIES OF DILATIONS

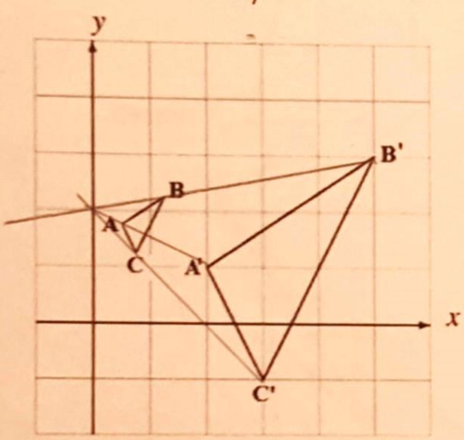
A **dilation** is a transformation that makes a figure larger or smaller than the original figure based on a ratio called a **scale factor**. These are the effects for the value of a scale factor.

Scale Factor is…	greater than 1	between 0 and 1	equals 1
Effect on the image	larger	smaller	same

The **center of dilation** is the point where the image has either become larger or smaller from the pre-image.

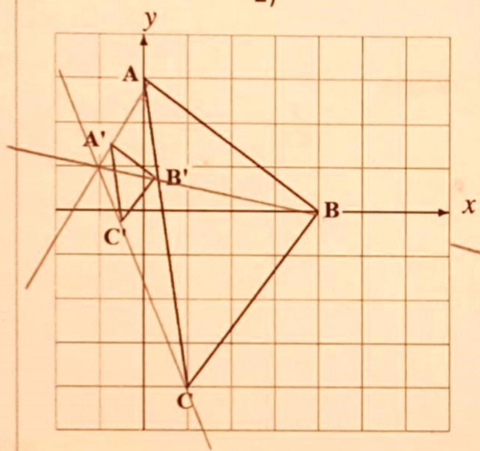
For #1-12, find the center and scale factor from the pre-image to the image.

1)



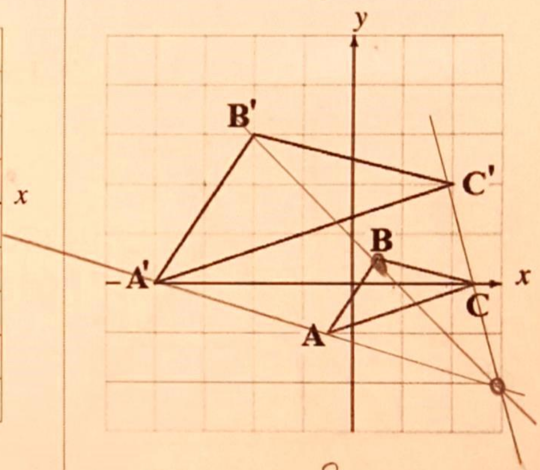
Center: (0, 2)

2)



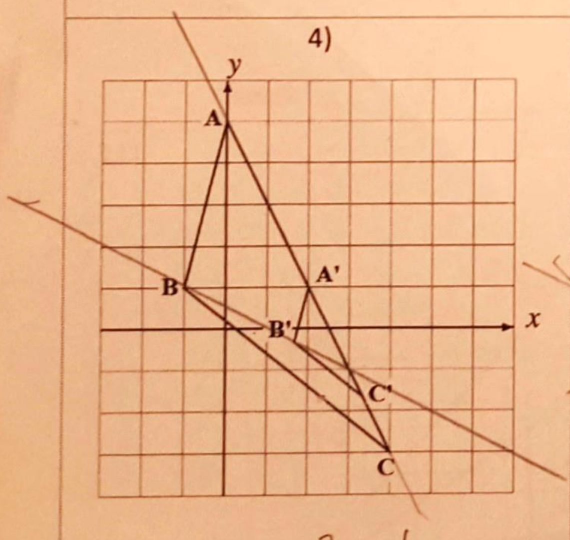
Center: (-1, 1)

3)



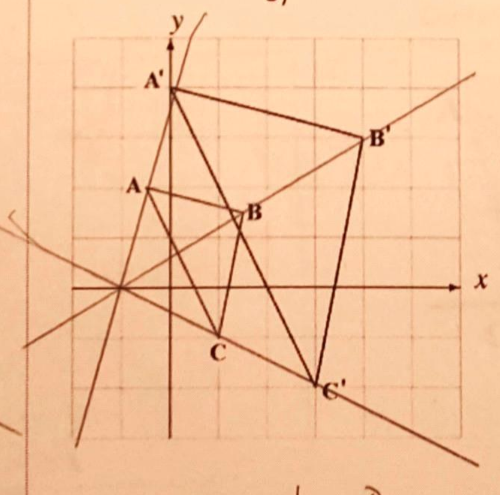
Center: (3, -2)

4)



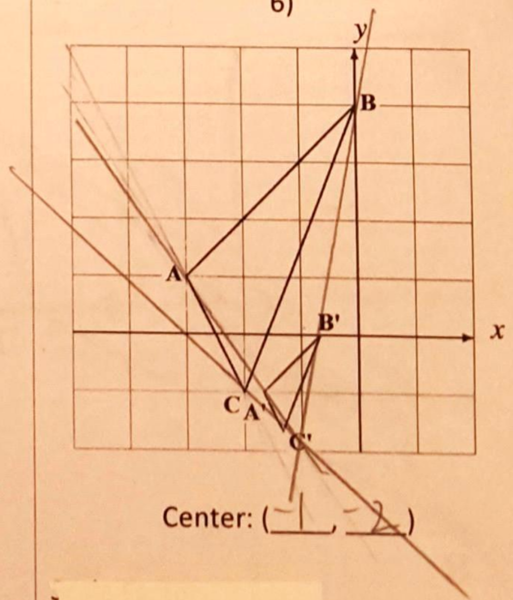
Center: (3, -1)

5)



Center: (-1, 0)

6)

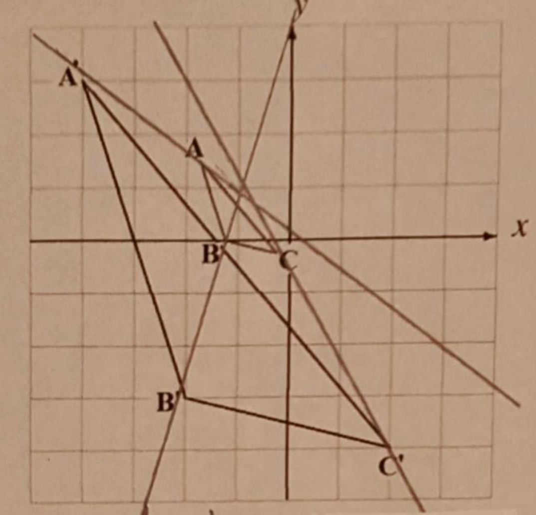


Center: (-1, -2)

Name: ______________________ Date: ______________

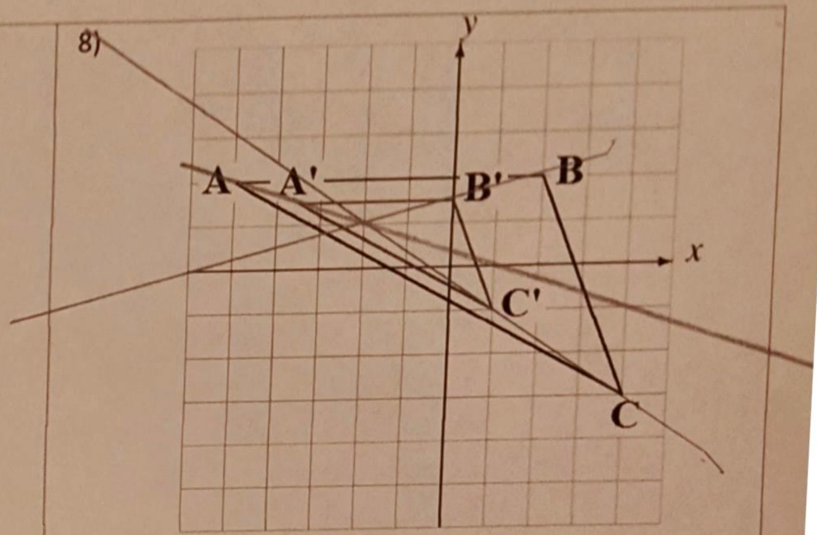
PROPERTIES OF DILATIONS

7)



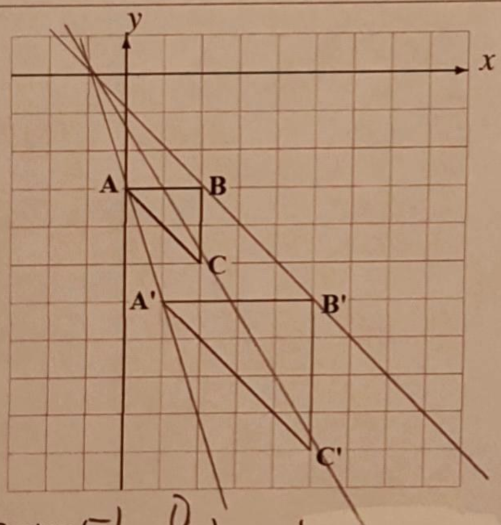
Center: (-1, 1)

8)



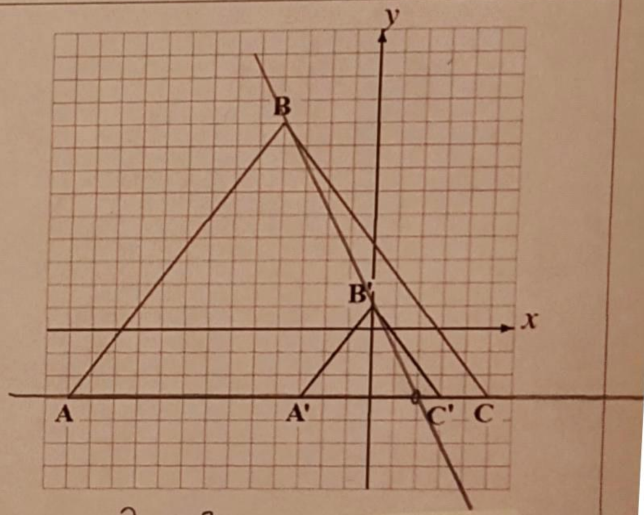
Center: (-2, 1)

9)



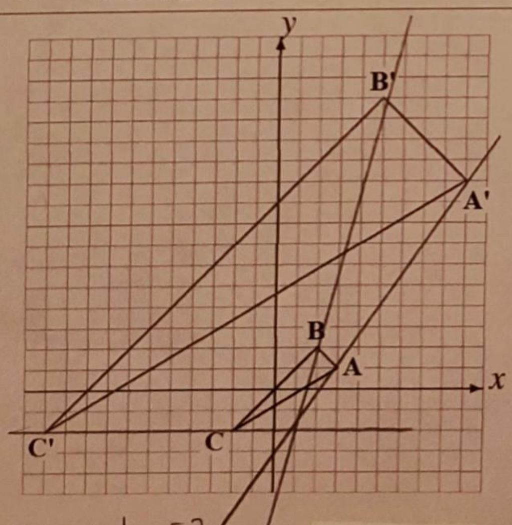
Center: (-1, 0)

10)



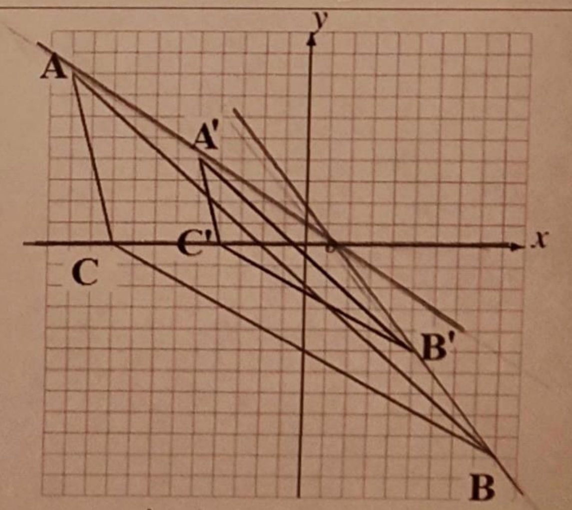
Center: (2, -3)

11)



Center: (1, -2)

12)



Center: (1, 0)