

Double Angles

Use a double-angle identity to find the exact value of each expression.

1) $\tan \theta = \frac{7}{24}$ and $0^\circ < \theta < 90^\circ$, find Q1

$x = 24$ $y = 7$ $r = 25$

$\sin 2\theta = 2\left(\frac{7}{25}\right)\left(\frac{24}{25}\right) = \frac{336}{625}$ Q1: All +

$\cos 2\theta = 2\left(\frac{24}{25}\right)^2 - 1 = \frac{527}{625}$

$\tan 2\theta = \frac{2\left(\frac{7}{24}\right)}{1 - \left(\frac{7}{24}\right)^2} = \frac{336}{527}$

3) $\cos \theta = -\frac{4}{5}$ and $90^\circ < \theta < 180^\circ$, find Q2

$x = 4$ $y = 3$ $r = 5$

$\sin 2\theta = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$ Q2: S +
C -

$\cos 2\theta = 2\left(-\frac{4}{5}\right)^2 - 1 = \frac{7}{25}$ + -

$\tan 2\theta = \frac{2\left(-\frac{3}{4}\right)}{1 - \left(-\frac{3}{4}\right)^2} = \frac{-24}{7}$

5) $\tan \theta = \frac{9}{40}$ and $180^\circ < \theta < 270^\circ$, find Q3

$x = 40$ $y = 9$ $r = 41$

$\sin 2\theta = 2\left(-\frac{9}{41}\right)\left(-\frac{40}{41}\right) = \frac{720}{1681}$ S sin -
C COS -

$\cos 2\theta = 2\left(\frac{40}{41}\right)^2 - 1 = \frac{904}{1681}$ tan +

$\tan 2\theta = \frac{2\left(\frac{9}{40}\right)}{1 - \left(\frac{9}{40}\right)^2} = \frac{720}{1681}$

2) $\sin \theta = -\frac{3}{5}$ and $270^\circ < \theta < 360^\circ$, find Q4

$x = 4$ $y = 3$ $r = 5$ Q4: sin -
cos +
tan -

$\sin 2\theta = 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = -\frac{24}{25}$

$\cos 2\theta = 1 - 2\left(-\frac{3}{5}\right)^2 = \frac{7}{25}$

$\tan 2\theta = \frac{2\left(-\frac{3}{4}\right)}{1 - \left(-\frac{3}{4}\right)^2} = \frac{-24}{7}$

4) $\cos \theta = -\frac{15}{17}$ and $90^\circ < \theta < 180^\circ$, find Q2

$x = 15$ $y = 8$ $r = 17$ Q2: sin +
cos -
tan -

$\sin 2\theta = 2\left(\frac{8}{17}\right)\left(-\frac{15}{17}\right) = -\frac{240}{289}$

$\cos 2\theta = 2\left(-\frac{15}{17}\right)^2 - 1 = \frac{161}{289}$

$\tan 2\theta = \frac{2\left(-\frac{8}{15}\right)}{1 - \left(-\frac{8}{15}\right)^2} = \frac{-240}{161}$

6) $\sin \theta = \frac{60}{61}$ and $90^\circ < \theta < 180^\circ$, find Q2

$x = 11$ $y = 60$ $r = 61$ Q2: sin +
cos -
tan -

$\sin 2\theta = 2\left(\frac{60}{61}\right)\left(-\frac{11}{61}\right) = -\frac{355}{1861}$

$\cos 2\theta = 1 - 2\left(\frac{60}{61}\right)^2 = -\frac{935}{1861}$

$\tan 2\theta = \frac{2\left(\frac{60}{11}\right)}{1 - \left(\frac{60}{11}\right)^2} = \frac{355}{1861}$

7) $\cot \theta = -\frac{15}{8}$ and $270^\circ < \theta < 360^\circ$, find Q4
 $x = 15$ $y = 8$ $r = 17$ sin -
cos +

$\sin 2\theta = 2\left(\frac{-8}{17}\right)\left(\frac{+15}{17}\right) = \frac{-240}{289}$ tan -

$\cos 2\theta = 2\left(\frac{15}{17}\right)^2 - 1 = \frac{161}{289}$

$\tan 2\theta = \frac{2\left(\frac{-8}{15}\right)}{1 - \left(\frac{-8}{15}\right)^2} = \frac{-240}{161}$

8) $\sec \theta = -\frac{25}{24}$ and $180^\circ < \theta < 270^\circ$, find Q3
 $x = 24$ $y = 7$ $r = 25$ sin -
cos -

$\sin 2\theta = 2\left(-\frac{7}{25}\right)\left(-\frac{24}{25}\right) = \frac{336}{625}$ tan +

$\cos 2\theta = 2\left(\frac{24}{25}\right)^2 - 1 = \frac{527}{625}$

$\tan 2\theta = \frac{2\left(\frac{7}{24}\right)}{1 - \left(\frac{7}{24}\right)^2} = \frac{336}{527}$

9) $\csc \theta = -\frac{25}{24}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find Q4
 $x = 7$ $y = 24$ $r = 25$ sin -
cos +

$\sin 2\theta = 2\left(-\frac{24}{25}\right)\left(\frac{+7}{25}\right) = \frac{-336}{625}$ tan -

$\cos 2\theta = 2\left(\frac{+7}{25}\right)^2 - 1 = \frac{-527}{625}$

$\tan 2\theta = \frac{2\left(\frac{24}{7}\right)}{1 - \left(-\frac{24}{7}\right)^2} = \frac{336}{527}$

10) $\sin \theta = -\frac{3}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find Q4
 $x = 4$ $y = 3$ $r = 5$ sin -
cos +

$\sin 2\theta = 2\left(-\frac{3}{5}\right)\left(\frac{+4}{5}\right) = \frac{-24}{25}$ tan -

$\cos 2\theta = 1 - 2\left(-\frac{3}{5}\right)^2 = \frac{7}{25}$

$\tan 2\theta = \frac{2\left(-\frac{3}{4}\right)}{1 - \left(-\frac{3}{4}\right)^2} = \frac{-24}{7}$

11) $\tan \theta = +\frac{8}{15}$ and $\pi < \theta < \frac{3\pi}{2}$, find Q3
 $x = 15$ $y = 8$ $r = 17$ sin -
cos -

$\sin 2\theta = 2\left(\frac{-8}{17}\right)\left(\frac{-15}{17}\right) = \frac{240}{289}$ tan +

$\cos 2\theta = 2\left(\frac{15}{17}\right)^2 - 1 = \frac{161}{289}$

$\tan 2\theta = \frac{2\left(\frac{+8}{15}\right)}{1 - \left(\frac{+8}{15}\right)^2} = \frac{240}{161}$

12) $\cos \theta = -\frac{24}{25}$ and $\frac{\pi}{2} < \theta < \pi$, find Q2
 $x = 24$ $y = 7$ $r = 25$ sin +
cos -

$\sin 2\theta = 2\left(\frac{7}{25}\right)\left(-\frac{24}{25}\right) = \frac{-336}{625}$ tan -

$\cos 2\theta = 2\left(\frac{-24}{25}\right)^2 - 1 = \frac{527}{625}$

$\tan 2\theta = \frac{2\left(\frac{-7}{24}\right)}{1 - \left(\frac{-7}{24}\right)^2} = \frac{-336}{527}$