Writing Equations of Ellipses

Things to remember

- The value of a = the length from the center to the vertex.
- The value of b = the length from the center to the covertex.
- The value of c = the length from the center to the foci.
- The vertices fall on the major axis whereas, the covertices fall on the minor axis.

<table>
<tr><th>What am I given?</th><th>EXAMPLE:</th><th>EXAMPLE:</th></tr>
<tr>
<td>Vertices and Covertices

Step 1 Find the center using the midpoint
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Step 2: Find the length of a.
a = center to vertex
Determine a^2

Step 3: Find the length of b.
b = center to covertex
Determine b^2

Step 4: Substitute the values of a^2, b^2, and (h,k) into the formula
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ or $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$</td>
<td>a Vertices: (-5,1) and (1,1) x changes
b Covertices: (-2,0) and (-2,2)
① center: $\left(\frac{-5+1}{2}, \frac{1+1}{2}\right) = (-2,1)$
② a = center to vertex
(-2,1), (-5,1)
a = 3
$a^2 = 9$
③ b = center to covertex
(-2,1) (-2,0)
b = 1
$b^2 = 1$
④ $\frac{(x+2)^2}{9} + \frac{(y-1)^2}{1} = 1$</td>
<td>a Vertices: (-8,10) and (-8,-2) y changes
b Covertices: (-4,4) and (-12,4)
① center: $\left(\frac{-8+-8}{2}, \frac{10+-2}{2}\right) = (-8,4)$
② a = center to vertex
(-8,4) to (-8,10)
a = 6
$a^2 = 36$
③ b = center to covertex
(-8,4) to (-4,4)
b = 4
$b^2 = 16$
④ $\frac{(x+8)^2}{16} + \frac{(y-4)^2}{36} = 1$</td>
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<tr>
<td>Vertices and Foci

Step 1 Find the center using the midpoint
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Step 2: Find the length of c.
c= center to foci
Determine c^2

Step 3: Find the length of a.
a= center to vertex
Determine a^2

Step 4: Use the formula $c^2 = a^2 - b^2$ to find the value of b^2 by substituting the values of a^2 and c^2 into the formula. Solve for b^2.

Step 5: Substitute the values of a^2, b^2, and (h,k) into the formula
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ or $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$</td>
<td>a Vertices: (3,-5) and (-7,-5) x changed
c Foci: (1,-5) and (-5,-5)
① center: $\left(\frac{3+-7}{2}, \frac{-5+-5}{2}\right) = (-2,-5)$
② c = center to foci
(-2,-5) to (1,-5)
c = 3
$c^2 = 9$
③ a = center to vertices
(-2,-5) to (3,-5)
a = 5
$a^2 = 25$
④ $c^2 = a^2 - b^2$
$9 = 25 - b^2$
$-16 = -b^2$
$\cancel{b^2 = 16}$
$\frac{(x+2)^2}{25} + \frac{(y+5)^2}{16} = 1$</td>
<td>a Vertices: (-8,14), (-8,-12) y changed
c Foci: (-8,13), (-8,-11)
① center: $\left(\frac{-8+-8}{2}, \frac{14+-12}{2}\right) = (-8,1)$
② c = center to foci
(-8,1) to (-8,13)
c = 12
$c^2 = 144$
③ a = center to vertices
(-8,1) to (-8,14)
a = 13
$a^2 = 169$
④ $c^2 = a^2 - b^2$
$144 = 169 - b^2$
$-25 = -b^2$
$b^2 = 25$
$\frac{(x+8)^2}{25} + \frac{(y-1)^2}{169} = 1$</td>
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