Hyperbola

- A Hyperbola is made up of 2 parabolas that are symmetrical
- The denominators of the equation determine how tall and wide the box is.
- The Vertices of a Hyperbola always lie on the parabolas which always go in the direction of the positive variable.
- The Foci points always lie on the inside of the parabolas.

With Hyperbolas, a^2 is always the first denominator!

Standard Form of an Ellipse

Center is (h,k).

a is the distance from the center to each vertex on the major axis

b is the distance from the center to each of the covertex

c is the distance from the center to each foci on the major axis

The rectangle with diagonals help you find the width of your graph.

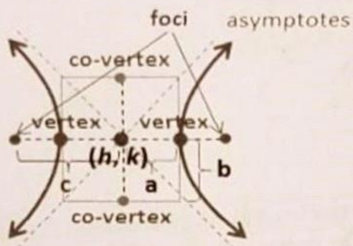
**Use the values of a and b to draw the central "invisible" Rectangles. Then draw lines through the diagonals.

Transverse axis: The axis that contains the vertices.

Conjugate axis: The axis that contains the covertices

If "x" is first:

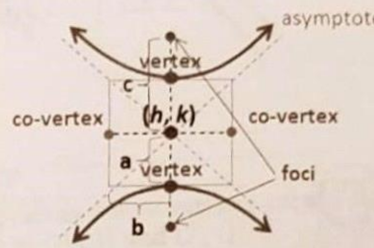
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



Vertices	$(h \pm a, k)$
Covertices	$(h, k \pm b)$
Asymptotes:	$y = k \pm \frac{b}{a}(x-h)$
Transverse axis	x-axis
Conjugate Axis	y-axis

If "y" is first

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$



Vertices	(h, k)
Covertices	$(h, k \pm a)$
Asymptotes:	$(h \pm b, k)$
Transverse axis	y-axis
Conjugate Axis	x-axis

x is first! so $a^2 = 25$

Graph the following:

$$\frac{(x-4)^2}{25} - \frac{(y+2)^2}{4} = 1$$

Center: $(4,2)$

$a^2 =$ 25 $a =$ 5

$b^2 =$ 4 $b =$ 2

Transverse axis: x

Vertices: $(4 \pm 5, 2)$ $(9,2), (-1,2)$

Co-Vertices: $(4, 2 \pm 2)$ $(4,4), (4,0)$

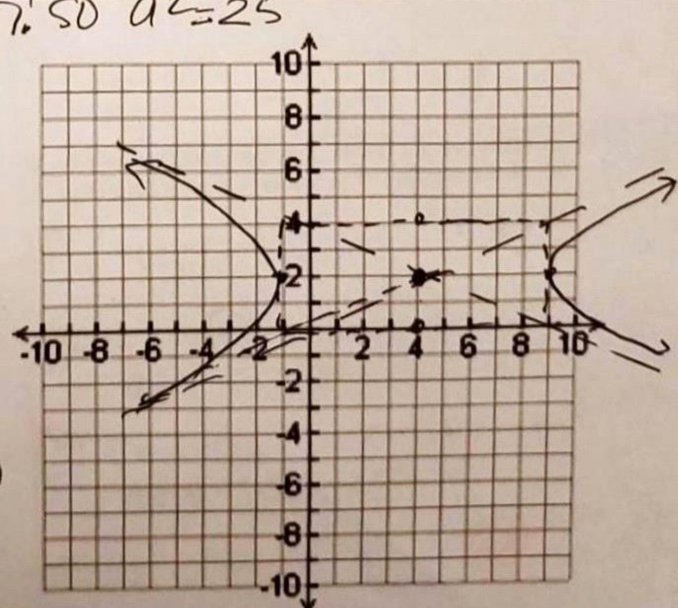
Foci Distance: $c^2 = a^2 + b^2$

$\sqrt{c^2} = \sqrt{25+4}$

$c = \pm\sqrt{29}$

Foci Points:

$(4 \pm \sqrt{29}, 2)$



y is first, so $a^2 = 25$

...e following:

$$\frac{(y+2)^2}{25} - \frac{(x-3)^2}{16} = 1$$

Center: (3, -2)

$a^2 =$ 25 a = 5

$b^2 =$ 16 b = 4

Transverse axis: y

Vertices: $(3, -2 \pm 5)$ $(3, -7), (3, 3)$

Co-Vertices: $(3 \pm 4, -2)$ $(7, -2), (-1, -2)$

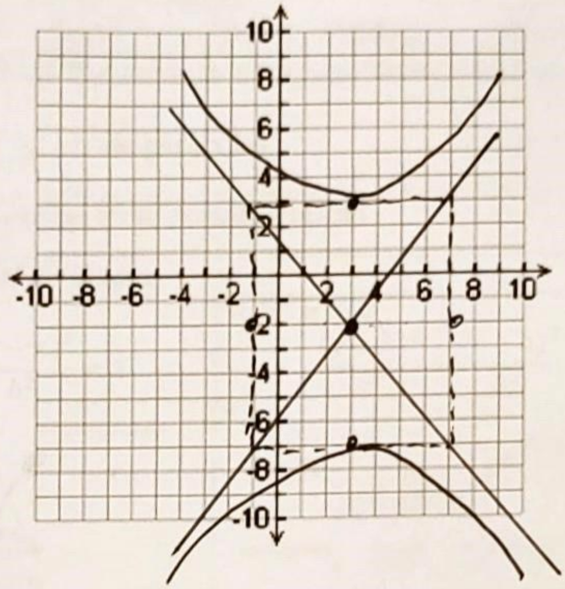
Foci Distance: $\cancel{c^2 = a^2 + b^2}$

$\sqrt{c^2} = \sqrt{25 + 16}$

$c = \pm\sqrt{41}$

Foci Points:

$(3, -2 \pm \sqrt{41})$



Writing Equations of a Hyperbola Given

1. Look to see what coordinates change.
   - ✓ If the x-coordinates change, the transverse axis will be horizontal (x is first and a^2 will be under x)
   - ✓ If the y-coordinates change, the transverse axis will be vertical (y is first and a^2 will be under y)

2. Find the center.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

3. Find the length of a.
   a= center to vertex
   Determine a^2

4. Find the length of c.
   c= center to foci
   Determine c^2

5. Use the formula $c^2 = a^2 + b^2$ to find the value of b^2 by substituting the values of a^2 and c^2 into the formula. Solve for b^2.

6. Substitute the values of a^2, b^2, and (h,k) into the formula.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Find the equation of a hyperbola whose vertices are at (-5, 1) and (1, 1) and whose foci are at (-6, 1) and (2, 1)

② $\left(\frac{-5+1}{2}, \frac{1+1}{2}\right) = (-2, 1)$

③ $a = (-2, 1) \rightarrow (-5, 1)$

$a = 3$ $a^2 = 9$

④ $c = (-2, 1)$ to $(-6, 1)$

$c = 4$ $c^2 = 16$

⑤ $c^2 = a^2 + b^2$

$16 = 9 + b^2$

-9 -9

$7 = b^2$

⑥ $\frac{(x+5)^2}{9} - \frac{(y-1)^2}{7} = 1$

Find the equation of a hyperbola whose vertices are at (-1, -1) and (-1, 7) and whose foci are at (-1, 8) and (-1, -2).

② $\left(\frac{-1+-1}{2}, \frac{-1+7}{2}\right) = (-1, 3)$

③ $a = (-1, 3) \rightarrow (-1, -1)$

$a = 4$ $a^2 = 16$

④ $c = (-1, 3) \rightarrow (-1, 8)$

$c = 5$ $c^2 = 25$

⑤ $c^2 = a^2 + b^2$

$25 = 16 + b^2$

-16 -16

$9 = b^2$

⑥ $\frac{(y+1)^2}{16} - \frac{(x-3)^2}{9} = 1$