Journal Entry #4 (refer to yellow sheet)

Find the coordinates of the image under the given transformation. Be sure to set up your matrix and write your answer as a matrix.

$(-y, x)$

Rotation 90° Counterclockwise

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -4 & 1 & -2 \\ -4 & -3 & -5 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 5 \\ -4 & 1 & -2 \end{bmatrix}$$

Jan 13-8:05 AM

The Identity Matrix

The *n x n* Identity matrix is the matrix that has 1's on the main diagonal (upper left to lower right) and 0's everyone where else.

The letter I is used to denote the identity matrix.

2 x 2 Identity Matrix

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3 x 3 Identity Matrix

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

IMPORTANT: *Not all matrices have an inverse. To determine if a matrix has an inverse, evaluate its determinant. If det A = 0, then the matrix does not have an inverse. Likewise, if det A ≠ 0, then the matrix has an inverse.*

The notation A^{-1} is used to denote the inverse matrix.

How to Find the Inverse Matrix

The inverse of matrix $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by:

- Type into Matrix.
- Then use x^{-1} [A] $^{-1}$ enter

$$\mathbf{A^{-1}} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \textbf{provided that } ad - cb \neq 0.$$

Example. *Find the inverse of* $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$.

$$\mathbf{A^{-1}} = \frac{1}{6-4}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix}$$

Example. *Find the inverse of each matrix.*

1) $\begin{bmatrix} -4 & 3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 3 & -4 \end{bmatrix}$

2) $\begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & -\frac{2}{3} \\ 0 & -1 \end{bmatrix}$

The inverse of a 3 x 3 matrix is difficult to compute by hand. A calculator will compute the inverse matrices.

Use the calculator to find the inverse of A = $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & 3 \end{bmatrix}$

$\mathbf{A^{-1}} =$ $\begin{bmatrix} -2/7 & 3/7 & 1/7 \\ -9/7 & 3/7 & 1/7 \\ -2/7 & -4/7 & 1/7 \end{bmatrix}$

Solving Matrix Equations using Inverses

Matrix Equations will be written in the form AX = B, where A and B are 2 by 2 matrices. To solve this type of equation, You must get it in the form: $\mathbf{X = A^{-1}B}$.

!!! ORDER MATTERS! On the right side, you MUST put the inverse first: $\boldsymbol{x = A^{-1}B}$

Solve the Matrix Equation. Be sure to write in the form $\mathbf{x = A^{-1}B}$. Use your calculator 😊

1) $\begin{bmatrix} -5 & -13 \\ 0 & 5 \end{bmatrix} X = \begin{bmatrix} 3 & 1 \\ -4 & 0 \end{bmatrix}$

$X = \begin{bmatrix} -5 & -13 \\ 0 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 3 & 1 \\ -4 & 0 \end{bmatrix}$

$\begin{bmatrix} 1.48 & -.2 \\ -.8 & 0 \end{bmatrix}$

2) $\begin{bmatrix} 5 & -1 \\ 8 & 2 \end{bmatrix} X = \begin{bmatrix} 17 & 20 \\ 26 & 20 \end{bmatrix}$

$X = \begin{bmatrix} 5 & -1 \\ 8 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 17 & 20 \\ 26 & 20 \end{bmatrix}$

$\begin{bmatrix} 10/3 & 10/3 \\ -1/3 & -10/3 \end{bmatrix}$