

Main Ideas/Questions

Notes

Trigonometric Identities

- A **trigonometric identity** is an equation involving trigonometric functions that is true for all values for which every expression in the equation is defined.
- The identities help to evaluate functions, simplify expressions, prove identities, and solve equations.

**Fundamental Trigonometric Identities**

$x^2 + y^2 = r^2$

$\sin \theta = \frac{y}{r}$     $\csc \theta = \frac{r}{y}$

$\cos \theta = \frac{x}{r}$     $\sec \theta = \frac{r}{x}$

$\tan \theta = \frac{y}{x}$     $\cot \theta = \frac{x}{y}$

**Quotient Identities**

$\tan \theta = \frac{\sin \theta}{\cos \theta}; \cos \theta \neq 0$        $\cot \theta = \frac{\cos \theta}{\sin \theta}; \sin \theta \neq 0$

**Reciprocal Identities**

$\sin \theta = \frac{1}{\csc \theta}; \csc \theta \neq 0$        $\csc \theta = \frac{1}{\sin \theta}; \sin \theta \neq 0$

$\cos \theta = \frac{1}{\sec \theta}; \sec \theta \neq 0$        $\sec \theta = \frac{1}{\cos \theta}; \cos \theta \neq 0$

$\tan \theta = \frac{1}{\cot \theta}; \cot \theta \neq 0$        $\cot \theta = \frac{1}{\tan \theta}; \tan \theta \neq 0$

**Pythagorean Identities**

$\cos^2 \theta + \sin^2 \theta = 1$        $\tan^2 \theta + 1 = \sec^2 \theta$        $\cot^2 \theta + 1 = \csc^2 \theta$

**Finding Trig Function Values**

Q1: All +

Q2: Sin, csc +

Q3: Tan, cot +

Q4: Cos, sec +

1. Find the exact value of  $\sin \theta$  if  $\cos \theta = -\frac{4}{7}$  and  $\frac{\pi}{2} < \theta < \pi$  Q2

$x = -4$     $y = \sqrt{33}$     $r = 7$

$x^2 + y^2 = r^2$   
 $(-4)^2 + y^2 = 7^2$   
 $16 + y^2 = 49$   
 $\sqrt{y^2} = \sqrt{49-16}$     $y = \sqrt{33}$

$\sin \theta = \frac{y}{r} = \frac{\sqrt{33}}{7}$        $\csc \theta = \frac{7\sqrt{33}}{33}$

$\cos \theta = -\frac{4}{7}$        $\sec \theta = -\frac{7}{4}$

$\tan \theta = -\frac{\sqrt{33}}{4}$        $\cot \theta = -\frac{4\sqrt{33}}{33}$

2. Find the exact value of  $\cos \theta$  if  $\cot \theta = \frac{2}{5}$  and  $180 < \theta < 270^\circ$  Q3

$x = 2$     $y = -5$     $r = \sqrt{29}$

$x^2 + y^2 = r^2$   
 $2^2 + (-5)^2 = r^2$   
 $4 + 25 = 12$   
 $\sqrt{29} = \sqrt{12}$     $r = \sqrt{29}$

$\cos \theta = \frac{x}{r} = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$

3. Find the exact value of  $\tan \theta$  if  $\csc \theta = -\frac{2}{1}$  and  $\frac{3\pi}{2} < \theta < 2\pi$  Q4

$x = \sqrt{3}$     $y = -1$     $r = 2$

$x^2 + y^2 = r^2$   
 $x^2 + (-1)^2 = 2^2$   
 $\sqrt{x^2} = \sqrt{9}$     $x = \sqrt{3}$

$\tan \theta = \frac{y}{x} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$