Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = Total Amount
P = Principal (how much you put in)
r = interest rate (%)
n (→) = # of times interest is compounded a year
t = time in years

Annually = 1
Monthly = 12
Weekly = 52
Semiannually = 2
Quarterly = 4
Daily = 365

Jack invests $2,804 in a savings account with a fixed annual interest rate of 7.52% compounded 4 times per year. What will the account balance be after 9 years?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$P = 2804$
$r = 7.52\% = .0752$
$n = 4$
$t = 9$

$2804\left(1 + \frac{.0752}{4}\right)^{4 \cdot 9}$

$5482.51

Stephanie invests $3,614 in a retirement account with a fixed annual interest rate of 8.68% compounded 12 times per year. How long will it take for the account balance to reach $15,722.68?

$15{,}722.68 = 3614\left(1 + \frac{.0868}{12}\right)^{12t}$

$\frac{15{,}722.68}{3614} = \frac{3614}{3614}\left(1 + \frac{.0868}{12}\right)^{12t}$

$4.35 = \left(1 + \frac{.0868}{12}\right)^{12t}$

$4.35 = (1.007)^{12t}$

$\log 4.35 = \log(1.007)^{12t}$

$\frac{\log 4.35}{\log 1.007} = 12t$

$210.76 = 12t$

$t \approx 18$ years

Compound Interest (using Logs) Notes

Examples:

1) You deposit $5000 in an account that yields 3.6% annual interest. Find the balance after 2 years if the interest is compounded with the given frequencies:

a) Semiannually: $n=2$

$5000\left(1+\frac{.036}{2}\right)^{2(2)}$

$5000\left(1+\frac{.036}{2}\right)^{4}$

$\$5369.83$

b) Quarterly: $n=4$

$5000\left(1+\frac{.036}{4}\right)^{4\cdot 2}$

$5000\left(1+\frac{.036}{4}\right)^{8}$

$\$5371.54$

2) You were charged 8.8% compounded monthly ($n=12$) on your credit card balance of $2500. If you did not make any payments on the card, how much would you owe in total after 1 year?

$n=12$

$t=1$

$P=2500$

$r=.088$

$2500\left(1+\frac{.088}{12}\right)^{12\cdot 1}$

$\$2729.09$

3) You put $1 into an account that yields 5% compounded daily ($n=365$). How much money will you have after 1 year?

$t=1$

$P=\$1$

$r=.05$

$n=365$

$1\left(1+\frac{.05}{365}\right)^{365\cdot 1}$

$\$1.05$

4) How long will it take for $500 to double if the interest rate is 3.5% and it's compounded monthly?

$500 doubled = 1000

$n=12$

$A=5000$

$P=500$

$r=.035$

$t=?$

$1000=500\left(1+\frac{.035}{12}\right)^{12t}$

$2=(1.003)^{12t}$

$\frac{\log 2}{\log 1.003}=12t$

$\frac{231.40}{12}$ $t\approx 19$ years

5) How long will it take for $1500 to grow into $4000 if it compounds quarterly at 5.7 %?

$P=1500$

$A=4000$

$r=.057$

$t=?$

$n=4$

$4000=1500\left(1+\frac{.057}{4}\right)^{4t}$

$4000=1500(1.014)^{4t}$

$2.67=(1.014)^{4t}$

$\frac{\log 2.67}{\log 1.014}=4t$

$70.6=4t$

$t\approx 18$ years