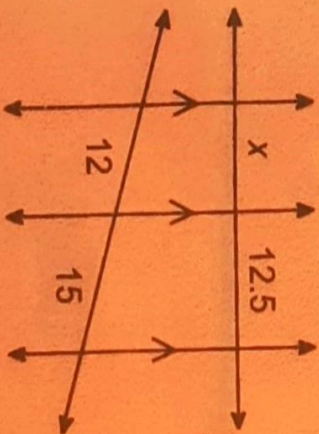


$$\frac{33}{x} = \frac{27}{18}$$

$$27x = 594$$

Find x.

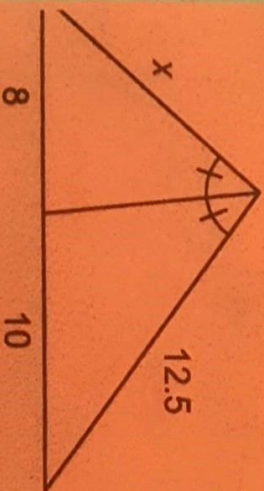


$$\frac{x}{12.5} = \frac{12}{15}$$

$$15x = 150$$

$$x = 10$$

Find x.



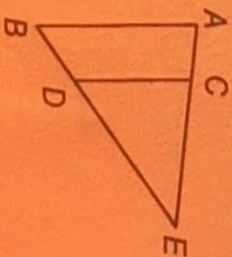
$$\frac{8}{10} = \frac{x}{12.5}$$

$$10x = 100$$

$$x = 10$$

### Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally. (The converse is also true.)



If  $\overline{AB} \parallel \overline{CD}$ , then

$$\frac{AC}{CE} = \frac{BD}{DE}$$

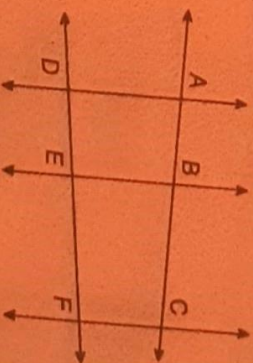
### Parallel Lines and Transversals

#### Proportionality Theorem

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

If  $\overline{AD} \parallel \overline{BE} \parallel \overline{CF}$ ,

then  $\frac{AB}{BC} = \frac{DE}{EF}$



### Triangle Angle Bisector Theorem

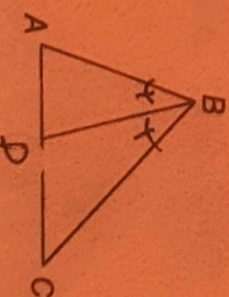
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

If  $\overline{BD}$  bisects  $\angle ABC$ ,

then  $\frac{AD}{DC} = \frac{AB}{BC}$



$$\frac{AD}{DC} = \frac{AB}{BC}$$



# PROPORTIONALITIES

## Triangle Proportionality

For #1-8, the given segments of the triangle from parallel lines are proportional. Find  $x$ . Show work.

<p>①</p> $\frac{10}{15} = \frac{14}{x}$ $10x = 210$ $x = 21$	<p>②</p> $\frac{6}{4} = \frac{x}{8}$ $4x = 48$ $x = 12$	<p>③</p> $\frac{20}{12} = \frac{25}{x}$ $20x = 300$ $x = 15$	<p>④</p> $\frac{36}{12} = \frac{48}{x}$ $36x = 576$ $x = 16$
<p>⑤</p> $\frac{10}{18} = \frac{15}{x}$ $10x = 270$ $x = 27$	<p>⑥</p> $\frac{7}{x} = \frac{4}{16}$ $4x = 112$ $x = 28$	<p>⑦</p> $\frac{25}{45} = \frac{20}{x}$ $25x = 900$ $x = 36$	<p>⑧</p> $\frac{10}{20} = \frac{16}{x}$ $10x = 320$ $x = 32$

## Triangle Proportionality Converse

For #1-8, write whether the given segments of the triangle are parallel or not. Show work.

<p>①</p> $\frac{24}{14} = \frac{36}{21} \neq \frac{12}{7} = \frac{12}{7}$ <p>Is <math>YM \parallel ET</math>? <u>Yes</u></p>	<p>②</p> $\frac{36}{45} = \frac{28}{42} \neq \frac{4}{5} = \frac{4}{5}$ <p>Is <math>TL \parallel BR</math>? <u>NO</u></p>	<p>③</p> $\frac{15}{36} = \frac{25}{55} \neq \frac{5}{12} = \frac{5}{12}$ <p>Is <math>TL \parallel CR</math>? <u>NO</u></p>	<p>④</p> $\frac{48}{56} = \frac{24}{63} \neq \frac{4}{7} = \frac{4}{7}$ <p>Is <math>AD \parallel GR</math>? <u>NO</u></p>
<p>⑤</p> $\frac{48}{8} = \frac{50}{18} \neq \frac{6}{5} = \frac{6}{5}$ <p>Is <math>OU \parallel SN</math>? <u>NO</u></p>	<p>⑥</p> $\frac{18}{24} = \frac{21}{28} \neq \frac{3}{4} = \frac{3}{4}$ <p>Is <math>FO \parallel GN</math>? <u>Yes</u></p>	<p>⑦</p> $\frac{15}{25} = \frac{18}{30} \neq \frac{3}{5} = \frac{3}{5}$ <p>Is <math>IN \parallel RD</math>? <u>Yes</u></p>	<p>⑧</p> $\frac{63}{35} = \frac{40}{25} \neq \frac{8}{5} = \frac{8}{5}$ <p>Is <math>RZ \parallel ON</math>? <u>NO</u></p>

## Parallel Line Proportionality

# PROPORTIONALITIES

For #1-8, the given segments formed by three parallel lines are proportional. Find x. Show work.

①

$$\frac{x+1}{7} = \frac{x+3}{8}$$

$$8x+8 = 7x+21$$

$$x = 13$$

②

$$\frac{12}{4} = \frac{x+6}{x-7}$$

$$12(x-7) = 4(x+6)$$

$$12x-84 = 4x+24$$

$$8x = 108 \quad x = 13.5$$

③

$$\frac{5}{6} = \frac{x-7}{x-5}$$

$$5x-25 = 6x-42$$

$$-x = -17 \quad x = 17$$

④

$$\frac{x+3}{7} = \frac{x}{6}$$

$$6x+18 = 7x$$

$$18 = x$$

⑤

$$\frac{12}{9} = \frac{x-3}{x-5}$$

$$12x-60 = 9x-27$$

$$3x = 33$$

$$x = 11$$

⑥

$$\frac{6}{3x+2} = \frac{3}{x+3}$$

$$6x+18 = 9x+6$$

$$-3x = -12$$

$$x = 4$$

⑦

$$\frac{x-7}{x+5} = \frac{6}{15}$$

$$15x-105 = 6x+30$$

$$+9x = 135$$

$$x = 15$$

⑧

$$\frac{2x-7}{10} = \frac{x+4}{8}$$

$$10x+40 = 16x-56$$

$$-6x = -96$$

$$x = 16$$

## Triangle Angle Bisector Proportionality

For #1-8, the given segments formed by an angle bisector are proportional. Find x. Show work.

①

$$\frac{15}{x} = \frac{30}{20}$$

$$30x = 300$$

$$x = 10$$

②

$$\frac{51}{34} = \frac{x}{40}$$

$$34x = 2040$$

$$x = 60$$

③

$$\frac{16}{36} = \frac{20}{x}$$

$$16x = 720$$

$$x = 45$$

④

$$\frac{8}{21} = \frac{x}{12}$$

$$12x = 168$$

$$x = 14$$

⑤

~~$$\frac{15}{18} = \frac{25}{x}$$~~

$$\frac{25}{15} = \frac{x}{18}$$

$$15x = 450$$

$$x = 30$$

⑥

$$\frac{24}{x} = \frac{36}{33}$$

$$36x = 792$$

$$x = 22$$

⑦

$$\frac{28}{26} = \frac{42}{x}$$

$$28x = 1092$$

$$x = 39$$

⑧

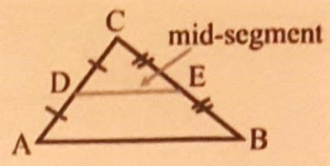
$$\frac{51}{68} = \frac{x}{76}$$

$$68x = 3876$$

$$x = 57$$

**Mid-segment Theorem:**

The mid-segment of a triangle (also called a midline) is a segment joining the midpoints of two sides of a triangle.



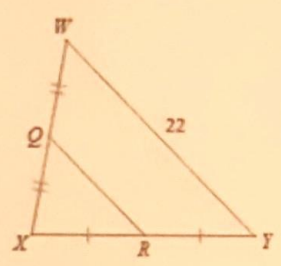
D is midpoint of  $\overline{AC}$ ; E is midpoint of  $\overline{BC}$   
 mid-segment  $\overline{DE}$ ;  $\overline{DE} \parallel \overline{AB}$ ;  $DE = \frac{1}{2}AB$

$2DE = AB$

**Examples:**

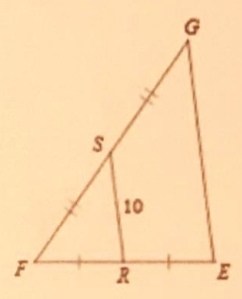
Find the missing length indicated.

1) Find  $RQ$



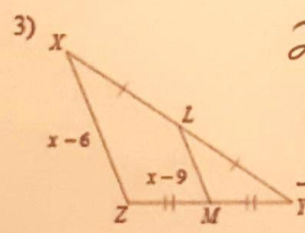
$RQ = 11$

2) Find  $EG$



$GE = 20$

Solve for  $x$ .

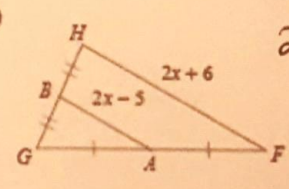


$$2(x-9) = x-6$$

$$2x-18 = x-6$$

$$\begin{array}{r} -x \quad -x \\ \hline x-18 = -6 \end{array}$$

$x = 12$



$$2(2x-5) = 2x+6$$

$$4x-10 = 2x+6$$

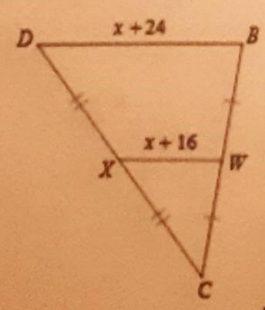
$$\begin{array}{r} -2x \quad -2x \\ \hline 2x-10 = 6 \end{array}$$

$$2x = 16$$

$x = 8$

Find the missing length indicated.

5) Find  $WX$



$WX = -8 + 16 = 8$

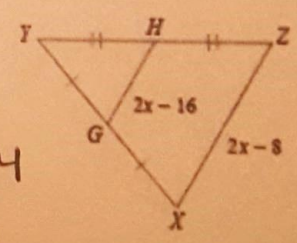
$$2(x+16) = x+24$$

$$2x+32 = x+24$$

$$\begin{array}{r} -x \quad -x \\ \hline x+32 = 24 \end{array}$$

$x = -8$

6) Find  $XZ$



$$2(2x-16) = 2x-8$$

$$4x-32 = 2x-8$$

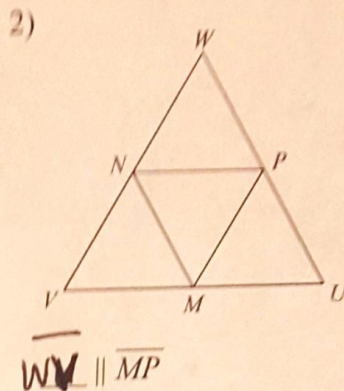
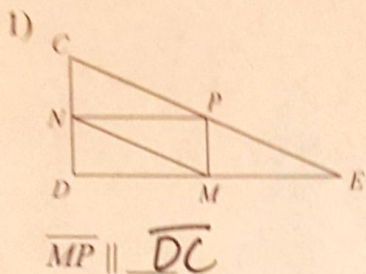
$$\begin{array}{r} -2x \quad -2x \\ \hline 2x-32 = -8 \end{array}$$

$$2x = 24$$

$x = 12$

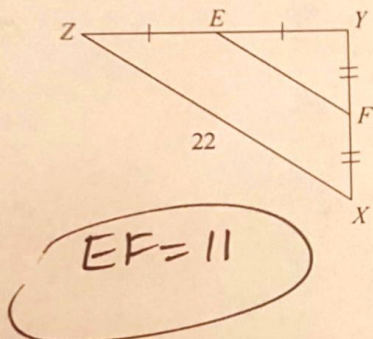
# Midsegment Practice

In each triangle, M, N, and P are the midpoints of the sides. Name a segment parallel to the one given.

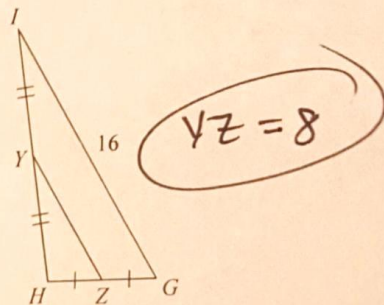


Find the missing length indicated.

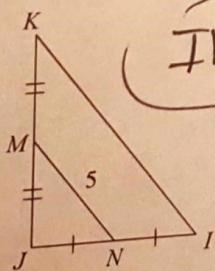
3) Find  $EF$



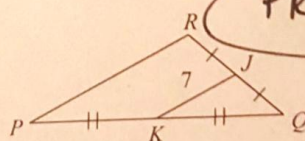
4) Find  $ZY$



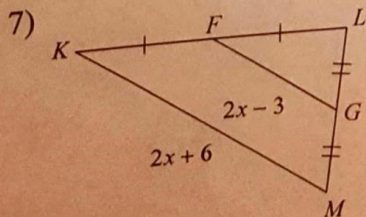
5) Find  $IK$



6) Find  $RP$



Solve for  $x$ .

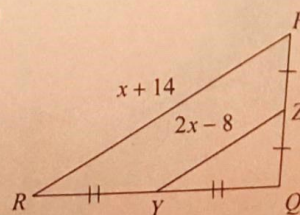


$$2(2x-3) = 2x+6$$

$$4x-6 = 2x+6$$

$$+2x = 12$$

$$x = 6$$



$$2(2x-8) = x+14$$

$$4x-16 = x+14$$

$$3x = 30$$

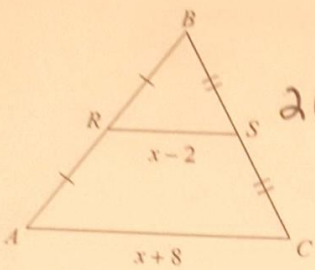
$$x = 10$$

Date:

Geometry

Name:

9)

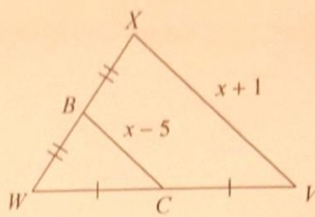


$$2(x-2) = x+8$$

$$2x-4 = x+8$$

$$x = 12$$

10)



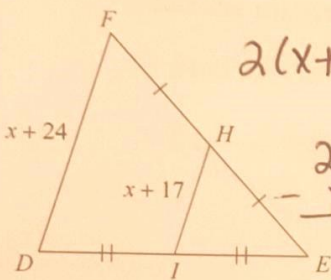
$$2(x-5) = x+1$$

$$2x-10 = x+1$$

$$x = 11$$

Find the missing length indicated.

11) Find  $FD$



$$2(x+17) = x+24$$

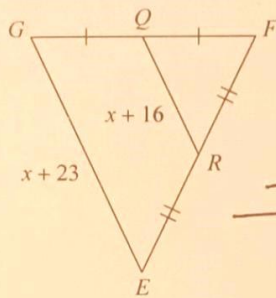
$$2x+34 = x+24$$

$$x+34 = 24$$

$$x = -10$$

$$FD = -10 + 24 = 14$$

12) Find  $GE$



$$2(x+16) = x+23$$

$$2x+32 = x+23$$

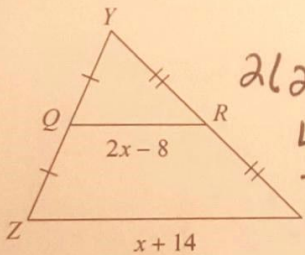
$$-x \quad -x$$

$$x+32 = 23$$

$$x = -9$$

$$GE = -9 + 23 = 14$$

13) Find  $ZX$



$$2(2x-8) = x+14$$

$$4x-16 = x+14$$

$$-x \quad -x$$

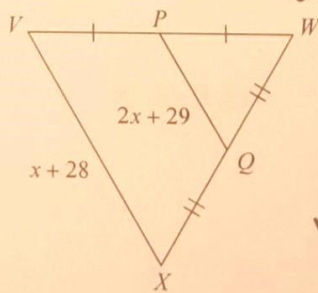
$$3x-16 = 14$$

$$3x = 30$$

$$x = 10$$

$$ZX = 10 + 14 = 24$$

14) Find  $VX$



$$2(2x+29) = x+28$$

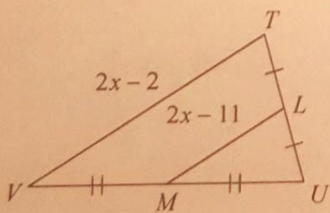
$$4x+58 = x+28$$

$$3x = -30$$

$$x = -10$$

$$VX = -10 + 28 = 18$$

15) Find  $LM$



$$2(2x-11) = 2x-2$$

$$4x-22 = 2x-2$$

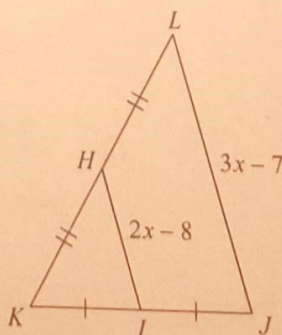
$$2x = 20$$

$$x = 10$$

$$LM = 2(10) - 11$$

$$= 9$$

16) Find  $IH$



$$2(2x-8) = 3x-7$$

$$4x-16 = 3x-7$$

$$x = 9$$

$$IH = 2(9) - 8$$

$$= 10$$