

$$1. \cos x \csc x$$

$$\frac{\cos x \cdot 1}{1 \sin x} = \frac{\cos x}{\sin x} = \tan x$$

$$2. \cos x \csc x \tan x$$

$$\frac{\cos x \cdot 1}{1 \sin x} \cdot \frac{\sin x}{\cos x} = 1$$

$$3. \tan x \cot x$$

$$\frac{\tan x \cdot 1}{1 \tan x} = 1$$

$$4. \sec x \cot x$$

$$\frac{1}{\sec x} \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin x} = \csc x$$

$$5. \sin x \cos x \cdot \sec x \cot x$$

$$\frac{\sin x \cos x}{1} \cdot \frac{1}{\sec x} \cdot \frac{\cos x}{\sin x} = \cos x$$

$$6. \cot x \sec x$$

$$\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} = \frac{1}{\sin x} = \cancel{\cos} \cdot \csc x$$

$$7. \sec x \cos x$$

$$\frac{1 \cdot \cos x}{\sec x} = 1$$

$$8. \cos x \tan x$$

$$\frac{\cos x \cdot \sin x}{1 \cos x} = \sin x$$

$$9. \sin x \sec x$$

$$\frac{\sin x \cdot 1}{1 \cos x} = \frac{\sin x}{\cos x} = \tan x$$

$$10) \csc x \sin x$$

$$\frac{1}{\sin x} \cdot \sin x = 1$$

$$11. \cos x \tan x + \sin x \cot x$$

$$\frac{\cos x \tan x}{1} + \sin x \frac{\cot x}{\sin x}$$

$$\sin x + \cos x$$

$$12. \sin x \sec x \cot x$$

$$\frac{\sin x}{\sin x} \cdot \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} = 1$$

$$13. \sin x (\csc x - \sin x)$$

$$\sin x \csc x - \sin^2 x$$

$$\frac{\sin x}{\sin x} \cdot \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$

$$1 - \sin^2 x$$

$$\cos^2 x$$

$$14. \cot x (\cos x \tan x + \sin x)$$

$$\cot x \cos x \tan x + \cot x \sin x$$

$$\frac{\cos x}{\sin x} \cdot \frac{\cos x}{1} \cdot \frac{\sin x}{\sin x} + \frac{\cos x}{\sin x} \cdot \frac{\sin x}{1}$$

$$= \cos x + \cos x$$

$$= 2 \cos x$$

$$15. \sec x \cot x - \cot x \cos x$$

$$\left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right) - \left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x}\right)$$

$$\frac{1 - \cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x}$$

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16. $\sin x \tan x - \csc x \tan x$

$$\frac{\sin x}{1} \cdot \frac{\sin x}{\cos x} - \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}$$

$$\frac{\sin^2 x - 1}{\cos^2 x} = -\frac{\cos^2 x}{\cos^2 x} = -1$$

17. $\tan x \div \sin x$

$$\tan x \cdot \frac{1}{\sin x}$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} = \frac{1}{\cos x} = \sec x$$

18. $\cot x \div \csc x$

$$\cot x \cdot \frac{1}{\csc x}$$

$$\frac{\cos x}{\sin x} \cdot \frac{\sin x}{1} = \cos x$$

19. $\sec x \div \csc x$

$$\sec x \cdot \frac{1}{\csc x}$$

$$\frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x} = \tan x$$

20. $\csc x \div \cot x$

$$\csc x \cdot \frac{1}{\cot x}$$

$$\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x} = \sec x$$

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$$21. \sec x \div \tan x$$

$$\sec x \cdot \frac{1}{\tan x}$$

$$\frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin x} = \csc x$$

$$22. \cot x \div \cos x$$

$$\cot x \cdot \frac{1}{\cos x}$$

$$\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} = \frac{1}{\sin x} = \csc x$$

$$23. \tan x \div \sin x$$

$$\tan x \cdot \frac{1}{\sin x}$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} = \frac{1}{\cos x} = \sec x$$

$$24. \csc^2 x \div \sec^2 x$$

$$\csc^2 x \cdot \frac{1}{\sec^2 x}$$

$$\frac{1}{\sin^2 x} \cdot \frac{\cos^2 x}{1} = \cancel{\sin^2 x} \frac{\cos^2 x}{\sin^2 x} = 1$$

$$25. \tan^2 x \div \sec^2 x$$

$$\tan^2 x \cdot \frac{1}{\sec^2 x}$$

$$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \sin^2 x$$

$$26. \sec x \cdot \sin x \div \tan x$$

$$\sec x \cdot \sin x \cdot \frac{1}{\tan x}$$

$$\frac{1}{\cos x} \cdot \cos x \cdot \frac{\sin x}{\tan x} = \frac{\cos x}{\sin x} = \cot x$$

$$27. \sec^2 x - 1 = \tan^2 x$$

$$28. 1 + \tan^2 x = \sec^2 x$$

$$29. 1 + \cot^2 x = \csc^2 x$$

$$30. 2(\csc^2 x - \cot^2 x)$$

$$2(1) = 2$$

$$31. \sin x (1 + \cot^2 x)$$

$$\begin{aligned} & \sin x + \sin x \cdot \cot^2 x \\ & \sin x + \cancel{\sin x \cdot \cos x} \\ & \quad \quad \quad \cancel{1} \quad \cancel{\sin x} \\ & \sin x + \cos x \end{aligned}$$

$$32. \cot^2 x (\sec^2 x - 1)$$

$$\cot^2 x (\tan^2 x)$$

$$\cancel{\cot^2 x} \cdot \frac{1}{\cancel{\cot^2 x}} = 1$$

$$33. \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}$$

$$\frac{1 - \cos^2 x}{\sin^2 x} \quad \frac{\sin^2 x}{\sin^2 x}$$

$$= 1$$

$$34. \frac{1 - \sin^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} = \cot^2 x$$

$$35. \cos x (\sec x - \cos x)$$

$$\cos x \sec x - \cos^2 x$$

$$\cos x \cdot \frac{1}{\cos x} - \cos^2 x$$

$$\frac{1 - \cos^2 x}{\sin^2 x}$$

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36. $\cot x(\tan x + \cot x)$

$$\cot x \tan x + \cot^2 x$$

$$\cot x \cdot \frac{1}{\sin x} + \cot^2 x$$

$$1 + \cot^2 x$$

$$= \csc^2 x$$

37. $\csc x - \cot x \cos x$

$$\frac{1}{\sin x} - \frac{\cos x \cdot \cos x}{\sin x}$$

$$\frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x}$$

$$= \sin x$$

38. $\frac{\csc x + \tan x}{1 + \tan^2 x}$

$$\frac{\csc x + \tan x}{\sec^2 x}$$

$$\csc x + \tan x \div \sec^2 x$$

$$\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\sec^2 x}$$

$$\frac{1 - \sin x}{\sin x \cos x} \cdot \frac{\cos^2 x}{\cos^2 x}$$

$$39. \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1}$$

$$\frac{1}{\tan^2 x + 1} = \frac{1}{\sec^2 x} = \boxed{\cos^2 x}$$

$$40. \frac{(1 + \sin x)(1 - \sin x)}{1 - \sin x + \sin x - \sin^2 x}$$

$$\frac{1 - \sin^2 x}{\cos^2 x}$$

$$\begin{aligned} 41. \frac{1 + \tan^2 x}{1 + \cot^2 x} &= \frac{\sec^2 x}{\csc^2 x} \\ &= \sec^2 x \div \csc^2 x \\ &= \frac{\sec^2 x}{1} \cdot \frac{1}{\csc^2 x} \\ &= \frac{1}{\cos^2 x} \cdot \sin^2 x \\ &= \frac{\sin^2 x}{\cos^2 x} \\ &= \boxed{\tan^2 x} \end{aligned}$$

$$42. \frac{1}{1 + \tan^2 x} + \frac{1}{1 + \cot^2 x}$$

$$\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x}$$

$$\begin{aligned} &= \cos^2 x + \sin^2 x \\ &= 1 \end{aligned}$$

$$43. \frac{\tan x + \cot x}{\sec^2 x}$$

$$\frac{\tan x + \cot x}{\sec^2 x} \div \sec^2 x$$
$$\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) \cdot \frac{1}{\sec^2 x}$$

$$\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) \cdot \cos^2 x$$

$$\cos^2 x \frac{\sin x}{\cos x} + \frac{\cos x \cos^2 x}{\sin x}$$

$$\cos x \sin x + \frac{\cos^3 x}{\sin x}$$

$$44. \frac{\sec^2 x}{\sec^2 x - 1} \quad \frac{\sec^2 x}{\tan^2 x}$$

$$\sec^2 x \div \tan^2 x$$

$$\frac{\sec^2 x}{1} \cdot \frac{1}{\tan^2 x}$$

$$\frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} = \csc^2 x$$

$$45) \frac{\sec x + \tan x}{\tan^2 x + 1}$$

$$\Rightarrow \sec x + \tan x \div \sec^2 x$$

$$\sec x \tan x = \frac{1}{\sec^2 x}$$

$$\begin{aligned} & \cancel{\sin} \frac{1}{\cancel{\cos x}} \cdot \frac{\sin x}{\cancel{\cos x}} \cdot \frac{\cos^2 x}{\boxed{\sin x}} \\ & \quad \end{aligned}$$

$$46. \csc^2 x \tan^2 x = 1$$

$$\begin{aligned} & \frac{1}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} = 1 \\ & \frac{1}{\cos^2 x} = 1 \end{aligned}$$

$$\frac{\sec^2 x}{\tan^2 x} = 1$$

$$47. \tan^2 x \sin^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{\sin^2 x}{1}$$

$$\frac{\sin^4 x}{\cos^2 x}$$

$$48. \csc^2 x - 1 = \cot^2 x$$

$$49. \sec^2 x - \tan^2 x = 1$$

$$50. \frac{\sec^2 x - 1}{\sin^2 x} = \frac{\tan^2 x}{\sin^2 x}$$

$$\tan^2 x \div \sin^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x}$$

$$\frac{1}{\cos^2 x} = \sec^2 x$$

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$$51) \sec^2 x (1 - \sin^2 x)$$

$$\sec^2 x (\cos^2 x)$$

$$\frac{1}{\cos^2 x} (\cos^2 x) = \boxed{1}$$

$$52. \frac{\tan^2 x + 1}{\tan^2 x} = \frac{\sec^2 x}{\tan^2 x}$$

$$\sec^2 x \div \tan^2 x$$

$$\sec^2 x \cdot \frac{1}{\tan^2 x}$$

$$\frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x}$$

$$\frac{1}{\sin^2 x} = \boxed{\csc^2 x}$$

$$53: \frac{1}{\tan^2 x + 1} = \frac{1}{\sec^2 x} = \cos^2 x$$

$$54) \frac{1 - \sec^2 x}{\tan^2 x} = \frac{1 - \frac{1}{\cos^2 x}}{\tan^2 x} = 1$$

$$55) \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1} = \frac{1}{\tan^2 x + 1} = \frac{1}{\sec^2 x} = \cos^2 x$$

$$56) \frac{\sin x}{1 - \cos^2 x} = \frac{\sin x}{\sin^2 x} = \frac{1}{\sin x} = \csc x$$