

Warm Up

The Ramos family bought 4 sandwiches and 3 salads. They spent \$24. Let x be the cost of a sandwich and y be the cost of a salad.

- a) Write a linear equation to represent this situation.

$$4x + 3y = 24$$

- b) Rewrite the equation in slope-intercept form.

$$\begin{array}{r} 4x + 3y = 24 \\ -4x \quad -4x \\ \hline 3y = -4x + 24 \\ \frac{3y}{3} = \frac{-4x}{3} + \frac{24}{3} \\ y = -\frac{4}{3}x + 8 \end{array}$$

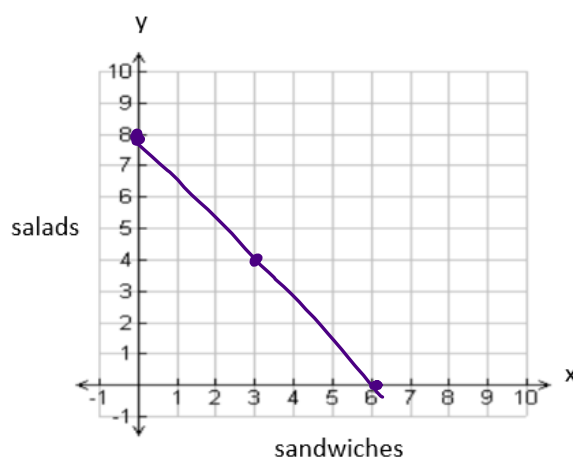
- c) Graph the equation.

- d) If they bought 3 sandwiches, how many salads did they buy?

4 salads

- e) List 3 combinations that could have been bought.

$$(0, 8), (3, 4), (6, 0)$$

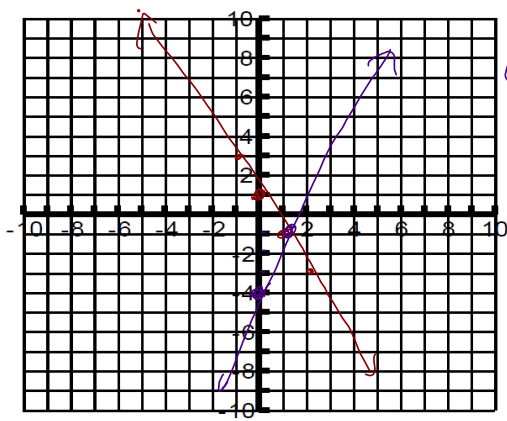


Special Types of Systems

Parallel

Lines have <u>Same</u> slopes	Have <u>same</u> slope, but different y-intercepts.	Have <u>Same</u> slope and same y-intercepts
Example: $y = -\frac{1}{2}x + 2$ $m = -\frac{1}{2}, b = 2$ $y = \frac{1}{2}x + 5$ $m = \frac{1}{2}, b = 5$ $(-4, 3)$	Example: $y = -2x + 4$ $m = -2, b = 4$ $y = -2x + 1$ $m = -2, b = 1$ 	Example: $y = 2x + 3$ $m = 2, b = 3$ $\frac{2y = 4x + 6}{2} = \frac{4x}{2} + \frac{6}{2}$ $y = 2x + 3$ $m = 2, b = 3$
Lines <u>intersect</u> . Exactly <u>one</u> solution. Algebraically: $x = \#$ or $y = \#$	Lines are <u>parallel</u> . There is <u>no</u> solution. Algebraically: $2 = 1$	Lines are the <u>same</u> . There are <u>infinitely</u> many solutions. Algebraically: $2 = 2$

Ways of Solving a System

Method	When do we use it?	Example:	
Graphing	When we want to find an approximate solution.	$y = -2x + 1$ $m = \frac{-2}{1}$ $b = 1$ $y = 3x - 4$ $m = \frac{3}{1}$ $b = -4$	
			
Substitution	When one (or both) equation(s) is solved for one variable.	$y = x + 3$ $2x - 4y = -12$ $2x - 4(x + 3) = -12$ $2x - 4x - 12 = -12$ $-2x - 12 = -12$ $+12 \quad +12$ $-2x = 0$ $x = 0$ $(0, 3)$ $y = x + 3$ $y = 0 + 3 = 3$	$y = x + 3$ $y = 2x - 4$ $x + 3 = 2x - 4$ $-2x \quad -2x$ $-x + 3 = -4$ $-3 \quad -3$ $x = 7$ $(7, 10)$ $y = x + 3$ $y = 7 + 3 = 10$

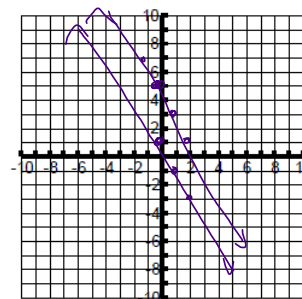
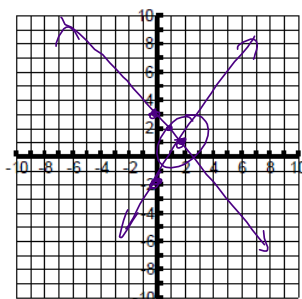
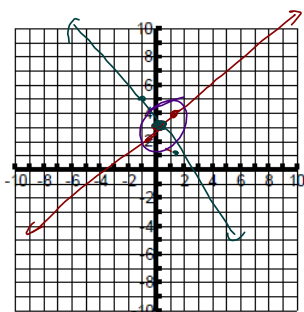
Solving Systems by Graphing Notes

Steps

1. Make sure each equation is in slope-intercept form.
2. Graph each equation on the same graph paper.
3. The point where the lines intersect is the solution.
If they don't intersect then there's no solution.
4. Check your solution algebraically!

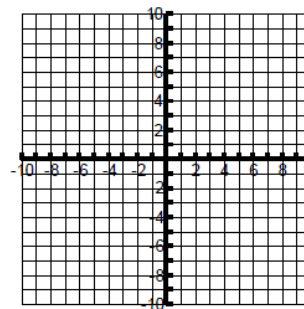
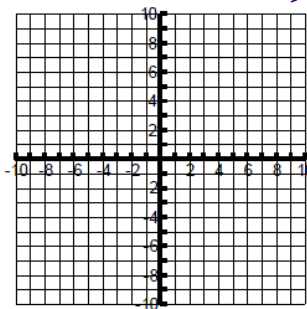
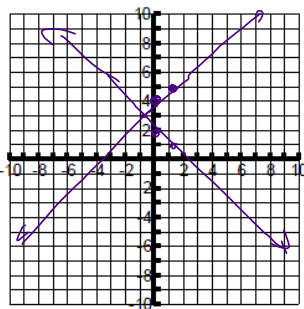
Examples:

$(0,3)$ 1. $\begin{cases} y = x + 3 & m=1, b=3 \\ y = -2x + 3 & m=-\frac{2}{1}, b=3 \end{cases}$ $(2,1)$ 2. $\begin{cases} y = -x + 3 & m=-\frac{1}{1}, b=3 \\ y = \frac{3}{2}x - 2 & m=\frac{3}{2}, b=-2 \end{cases}$ $\frac{NO}{SD!}$ 3. $\begin{cases} y = -2x + 5 & m=-\frac{2}{1}, b=5 \\ y = -2x + 1 & m=-\frac{2}{1}, b=1 \end{cases}$



$(-1,3)$ 4. $\begin{cases} y = x + 4 & m=\frac{1}{1}, b=4 \\ y = -x + 2 & m=-\frac{1}{1}, b=2 \end{cases}$ $(-3,1)$ 5. $\begin{cases} y = -x - 2 & m=-\frac{1}{1}, b=-2 \\ y = \frac{2}{3}x + 3 & m=\frac{2}{3}, b=3 \end{cases}$

$(-2,5)$ 6. $\begin{cases} y = 5 & m=0, b=5 \\ y = -2x + 1 & m=-\frac{2}{1}, b=1 \end{cases}$



Example In each of the following systems determine if the given point is a solution.

7. $\begin{cases} x + y = 9 \\ -2x + y = -3 \end{cases}$ $(4, -2)$

$4 + (-2) \neq 9$ NO!

8. $\begin{cases} 2x + y = -4 \\ 5x + 3y = -6 \end{cases}$ $(6, 5)$

$2(6) + 5 \neq -4$ NO

I. Determine if (2, 1) is a solution to the following systems: Answer yes or no!

$$\begin{aligned} x - y &= 1 \\ 1) \quad 3x + y &= -5 \end{aligned}$$

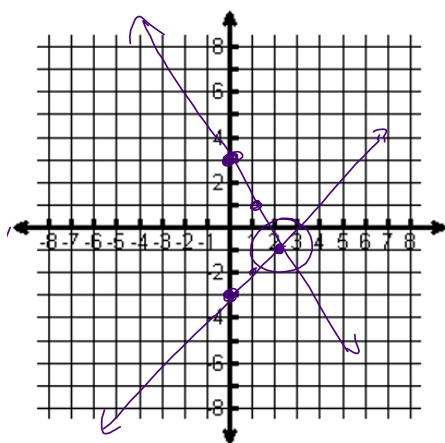
$$\begin{aligned} 2 - 1 &= 1 \quad \checkmark \\ 3(2) + 1 &= -5 \quad \times \\ \text{NO} \end{aligned}$$

$$\begin{aligned} -4x + 3y &= -5 \\ 2) \quad -x - y &= -3 \end{aligned}$$

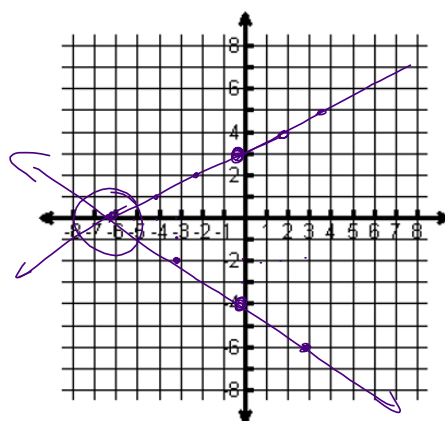
$$\begin{aligned} -4(2) + 3(1) &= -5 \quad \checkmark \\ -2 - 1 &= -3 \quad \checkmark \\ \text{yes} \end{aligned}$$

II. For 3 – 6, solve each system graphically. Write your solution in the blank provided.

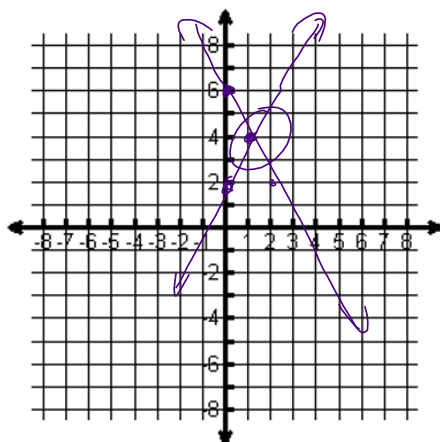
$$\begin{aligned} (2, -1) \quad 3) \quad y &= -2x + 3 \quad m = \frac{-2}{1} \quad b = 3 \\ y &= x - 3 \quad m = \frac{1}{1} \quad b = -3 \end{aligned}$$



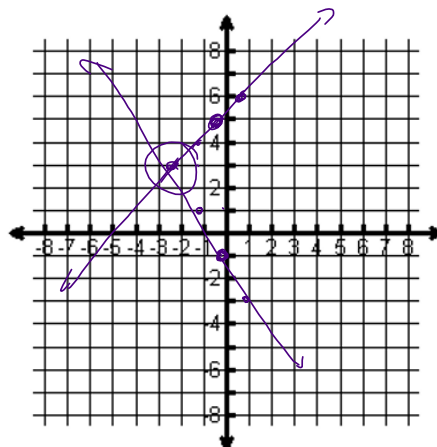
$$\begin{aligned} (-6, 0) \quad 4) \quad y &= \frac{1}{2}x + 3 \quad m = \frac{1}{2} \quad b = 3 \\ y &= -\frac{2}{3}x - 4 \quad m = \frac{-2}{3} \quad b = -4 \end{aligned}$$



$$\begin{aligned} (1, 4) \quad 5) \quad y &= -2x + 6 \quad m = \frac{-2}{1} \quad b = 6 \\ y &= 2x + 2 \quad m = \frac{2}{1} \quad b = 2 \end{aligned}$$



$$\begin{aligned} (-2, 3) \quad 6) \quad y &= x + 5 \quad m = \frac{1}{1} \quad b = 5 \\ y &= -2x - 1 \quad m = \frac{-2}{1} \quad b = -1 \end{aligned}$$



Solving Systems by Substitution Notes

Steps

1. One equation will have either x or y by itself or can be solved for x or y easily.
2. Substitute the expression from Step 1 into the other equation and solve for the other variable.
3. Substitute the value from Step 2 into the equation from Step 1 and solve.
4. Your solution is the ordered pair formed by x & y.
5. Check the solution in each of the original equations.

Examples:

<p>1. $x = -4$ $3x + 2y = 20$ $3(-4) + 2y = 20$ $-12 + 2y = 20$ $+12 \quad +12$ $2y = 32$ $y = 16$</p> <p>$(-4, 16)$</p>	<p>2. $y = x - 1$ $x + y = 3$ $x + x - 1 = 3$ $2x - 1 = 3$ $2x = 4$ $x = 2$</p> <p>$y = 2 - 1$ $y = 1$ $(2, 1)$</p>
<p>3. $3x + 2y = -12$ $y = x - 1$ $3x + 2(x - 1) = -12$ $3x + 2x - 2 = -12$ $5x - 2 = -12$ $5x = -10$ $x = -2$</p> <p>$y = 2 - 1$ $y = -3$ $(-2, -3)$</p>	<p>4. $x = \frac{1}{2}y - 3$ $4x - y = 10$ $4(\frac{1}{2}y - 3) - y = 10$ $2y - 12 - y = 10$ $y - 12 = 10$ $y = 22$</p> <p>$x = \frac{1}{2}(22) - 3$ $x = 11 - 3 = 8$ $(8, 22)$</p>
<p>5. $x = -5y + 4$ $3x + 15y = -1$ $3(-5y + 4) + 15y = -1$ $-15y + 12 + 15y = -1$ $12 = -1$ NO SOL.</p>	<p>6. $2x - 5y = 29$ $x = 4y + 8$ $2(4y + 8) - 5y = 29$ $-8y + 16 - 5y = 29$ $-13y + 16 = 29$ $-13y = 13$ $y = -1$</p> <p>$x = 4(-1) + 8$ $4 + 8 = 12$ $(12, -1)$</p>
<p>7. $x = 5y + 10$ $2x - 10y = 20$ $2(5y + 10) - 10y = 20$ $10y + 20 - 10y = 20$ $20 = 20$ Inf. Many Sol.</p>	<p>8. $x = 6y + 18$ $2x - 3y = -24$ $2(6y + 18) - 3y = -24$ $-12y + 36 - 3y = -24$ $-15y + 36 = -24$ $-15y = -60$ $y = 4$</p> <p>$x = -6(4) + 18$ $-24 + 18 = -6$ $(-6, 4)$</p>

III. Solve each linear system using substitution.

$$(5, 9) \quad 7) \quad \begin{cases} y = x + 4 \\ 2x + y = 19 \end{cases}$$

$$2x + x + 4 = 19$$

$$3x + 4 = 19$$

$$3x = 15$$

$$(x = 5)$$

$$y = 5 + 4$$

$$(y = 9)$$

$$(5, 9)$$

$$(-5, 18) \quad 8) \quad \begin{cases} y = -3x + 3 \\ 7x + 2y = 1 \end{cases}$$

$$7x + 2(-3x + 3) = 1$$

$$7x - 6x + 6 = 1$$

$$x + 6 = 1$$

$$(x = -5)$$

$$y = -3(-5) + 3$$

$$(y = 18)$$

$$y = (-5, 18)$$

$$(-\frac{1}{2}, 2) \quad 9) \quad \begin{cases} y = 2x + 3 \\ 2x + 3y = 5 \end{cases}$$

$$2x + 3(2x + 3) = 5$$

$$2x + 6x + 9 = 5$$

$$8x + 9 = 5$$

$$8x = -4$$

$$x = -4/8 = (-\frac{1}{2})$$

$$y = 2(-\frac{1}{2}) + 3$$

$$(y = -1 + 3 = 2)$$

$$(-\frac{1}{2}, 2)$$

$$(7, 10) \quad 10) \quad \begin{cases} y = 2x - 4 \\ 3x - 2y = 1 \end{cases}$$

$$3x - 2y = 1$$

$$3x - 2(2x - 4) = 1$$

$$3x - 4x + 8 = 1$$

$$-x + 8 = 1$$

$$-x = -7$$

$$y = 2(7) - 4$$

$$(y = 10)$$

$$(7, 10)$$

$$(6, 2) \quad 11) \quad \begin{cases} y = x - 4 \\ 4x + y = 26 \end{cases}$$

$$4x + x - 4 = 26$$

$$5x - 4 = 26$$

$$5x = 30$$

$$(x = 6)$$

$$y = 6 - 4$$

$$(y = 2)$$

$$(6, 2)$$

$$(5, 9) \quad 12) \quad \begin{cases} y = x + 4 \\ 2x + y = 19 \end{cases}$$

$$2x + x + 4 = 19$$

$$3x + 4 = 19$$

$$3x = 15$$

$$(x = 5)$$

$$y = 5 + 4$$

$$(y = 9)$$

$$(5, 9)$$

$$(2, 9) \quad 13) \quad \begin{cases} y = x + 7 \\ 2x + 3y = 31 \end{cases}$$

$$2x + 3(x + 7) = 31$$

$$2x + 3x + 21 = 31$$

$$5x + 21 = 31$$

$$5x = 10$$

$$(x = 2)$$

$$y = 2 + 7$$

$$(y = 9)$$

$$(2, 9)$$

$$(1, 2) \quad 14) \quad \begin{cases} y = -2x + 4 \\ -x + y = 1 \end{cases}$$

$$-x + -2x + 4 = 1$$

$$-3x + 4 = 1$$

$$-3x = -3$$

$$(x = 1)$$

$$y = -2(1) + 4$$

$$y = -2 + 4 = 2$$

$$(y = 2)$$

$$(1, 2)$$

Summary

Error Analysis: Find the mistake and correct it.

Solve the system by substitution:

$$y = -3x + 9$$

$$4x + 2y = 6$$

$$\text{Step 1: } 4x + 2(-3x + 9) = 6$$

$$4x - 6x + 18 = 6$$

$$-2x + 18 = 6$$

$$\underline{-18 = -18}$$

$$-2x = -12$$

$$x = 6$$

$$\text{Step 2: } y = -3x + 9$$

$$6 = -3x + 9$$

$$\underline{-9 \quad -9}$$

$$-3 = -3x$$

$$1 = x$$