Unit 5 Study Guide

Name _____ S _____

1. What information is needed to show that a parallelogram is a rectangle?

   diagonals are congruent

2. What information is needed to prove a quadrilateral is a parallelogram?

   diagonals bisect

3. What information is needed to show that a parallelogram is a square?

   diagonals congruent, perpendicular

4. Parallel lines have **same** slope and perpendicular lines have **opposite reciprocal** slopes.

5. Find the midpoint of the line segment containing

   A(5,-8) and B(-7,-4)

   $x_1 \; u_1 \qquad x_2 \; u_2$

   Midpoint of AB: $\left(\dfrac{-7+5}{2}, \dfrac{-4+-8}{2}\right)$

   $\left(-\dfrac{2}{2}, -\dfrac{12}{2}\right)$

   $(-1, -6)$

6. If a line segment needs to be portioned by a 3:4 ratio, what is the fraction that would be used to find the point?

   $\dfrac{3}{7}$

7. Identify the slope of each line and then determine if the lines are parallel, perpendicular, or neither.

   a) $y = -1/4x + 8$

      $y = 4x - 3$ **Perp.**

   b) $y = 1/2x + 5$

      $y = 1/2x - 9$ **Parallel**

   c) $y = 3x + 1$

      $y = -3x - 2$ **Neither**

Write the equation of the lines below in slope-intercept form: $y = mx + b$.

8. Through $(-4, 5)$ and parallel to $y = -\dfrac{3}{2}x - 5$.

   $m = -3/2$

   $m_{//} = -3/2$

   $y - y_1 = m(x - x_1)$

   $y - 5 = -\dfrac{3}{2}(x+4)$

   $y - 5 = -\dfrac{3}{2}x - 6$

   $\boxed{y = -\dfrac{3}{2}x - 1}$

9. Through $(4, 1)$ and perpendicular to $y = -2x - 2$
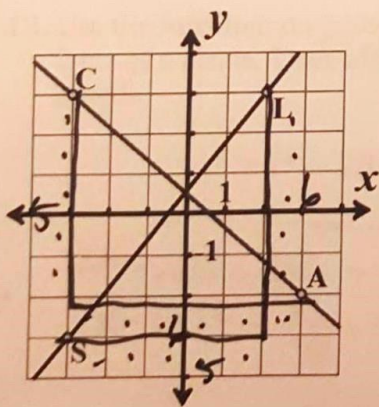
   $m = -2.$

   $m_{\perp} = -1/2$

   $y - y_1 = m(x - x_1)$

   $y - 1 = -\dfrac{1}{2}(x - 4)$

   $y - 1 = -\dfrac{1}{2}x + 2$

   $\boxed{y = \dfrac{1}{2}x + 3}$

10. Use the graph below of the two intersecting lines to explain how you can use the slope of each line to prove that they are perpendicular.
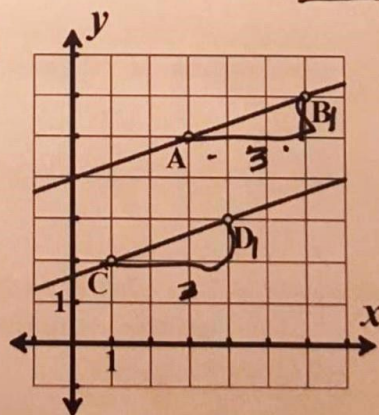
    opposite reciprocal

    rise / run

    Slope of $\overline{AC}$: $-5/6$

    Slope of $\overline{LS}$: $6/5$

    Slopes are **opposite reciprocal, so they are perpendicular lines**

11. Use the graph below of the non-intersecting lines to explain how you can use the slope of each line to prove that they are parallel.

    same slope

    Slope of $\overline{AB}$: $1/3$

    Slope of $\overline{CD}$: $1/3$

    Slopes are **the same, so they are parallel lines**

12. Given the points $P(2, -1)$ & $Q(-9, -6)$, what are the coordinates of the point on the directed line segment $\overline{PQ}$ that partitions $\overline{PQ}$ into the ratio

$x_1 \quad y_1 \qquad x_2 \quad y_2$

$3:2 = \frac{3}{5}$

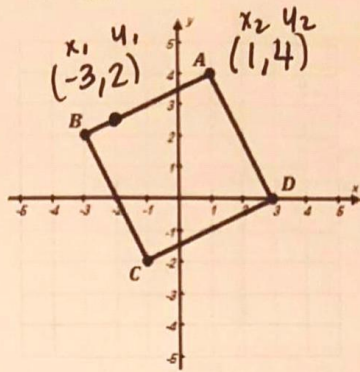$\Delta x = x_2 - x_1 = -9 - 2 = -11 \left(\frac{3}{5}\right) = -6.6$

$\Delta y = y_2 - y_1 = -6 - 1 = -5 \left(\frac{3}{5}\right) = -3$

$P(2, -1)$

$+ -6.6, -3$

$\boxed{(-4.6, -4)}$

13. Given the graph below, find the coordinates of $\overline{BA}$ if the ratio AD 1:3.

$x_1 \quad y_1 \qquad x_2 \quad y_2$
$(-3, 2) \qquad (1, 4)$

$1:3 = 1/4$

$\Delta x = x_2 - x_1 = 1 - -3 = 4\left(\frac{1}{4}\right) = 1$

$\Delta y = y_2 - y_1 = 4 - 2 = 2\left(\frac{1}{4}\right) = .5$

$B(-3, 2)$

$+ 1, .5$

$\boxed{-2, 2.5}$

14. Which point is on a circle with a center of $(3, -9)$ and a radius of 5? *(Hint: Use $(x-h)^2 + (y-k)^2 = r^2$)*

A. $(-6, 5)$ $\quad (-6-3)^2 + (5--9)^2 \overset{?}{=} 5^2$

B. $(-1, 6)$ $\quad (-1-3)^2 + (6--9)^2 \overset{?}{=} 5^2$

C. $(1, 6)$ $\quad (1-3)^2 + (6--9)^2 \overset{?}{=} 5^2$

D. $(6, -5)$ $\quad (6-3)^2 + (-5--9)^2 \overset{?}{=} 5^2$

15. What is the equation of the circle given the Center: $(4, 8)$, $diameter = 11$

$\dfrac{diameter}{2} = rad$

$\dfrac{5.5}{}$

5.5

Equation: $(x - 4)^2 + (y - 8)^2 = 5.5^2$

$(x-4)^2 + (y-8)^2 = 30.25$

16. Identify the center and radius of the circle whose equation is given below.

$(x + 10)^2 + (y - 7)^2 = 9$

Center: $(-10, 7)$ radius $= \sqrt{9} = 3$

17. Given graph below, identify the center, radius and write the equation of the circle.
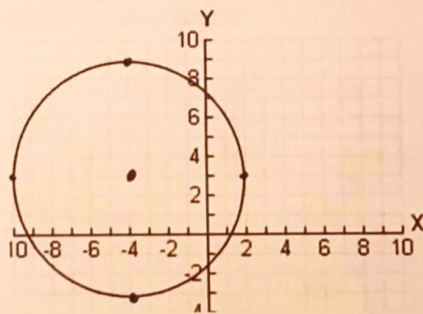
Center: $(-4, 3)$ radius $= 6$

Equation: $(x + 4)^2 + (y - 3)^2 = 6^2$ $\quad (x+4)^2 + (y-3)^2 = 36$

18. Use the information provided to write the standard form of a circle. Then idenify the center and radius length.

$x^2 + y^2 - 20x + 2y + 76 = 0$
$\quad\quad\quad\quad\quad -76 \quad -76$

$x^2 - 20x + 100 + y^2 + 2y + 1 = -76 + 100 + 1$

$(x - 10)^2 + (y + 1)^2 = 25$

$\frac{2}{2} = 10$

$\frac{2}{2} = 1$

Center: $(10, -1)$ radius $= 5$

Equation: $(x - 10)^2 + (y + 1)^2 = 5^2$

19. What is the length of the radius of the circle whose diameter has endpoints of $(-3, 2)$ and $(1, -4)$

$x_1 \quad y_1 \qquad x_2 \quad y_2$

Step 1) Find midpoint: $\left(\dfrac{-3 + 1}{2}, \dfrac{2 + -4}{2}\right)$ $\quad (-1, -1)$

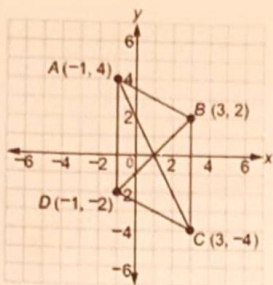Step 2) Take midpoint and one of endpoints and use distance formula. $\quad (-1, -1), (-3, 2)$

$= \sqrt{(-3 - -1)^2 + (2 - -1)^2}$

$= \sqrt{(-2)^2 + (3)^2}$

$= \sqrt{(4) + (9)} = \sqrt{13}$

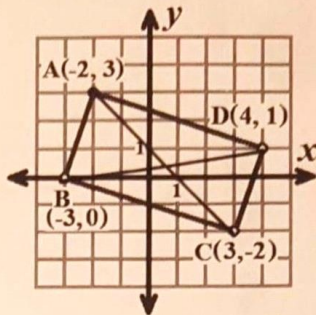$r = \sqrt{13}$ or $r \approx 3.61$

20. Prove that quadrilateral ABCD shown below is quadrilateral by showing the diagonals bisect each other at (1,0).

Midpoint of AC: $(\frac{-1+3}{2}, \frac{4+4}{2}) = (\frac{2}{2}, \frac{0}{2})$

$(1, 0)$ ✓

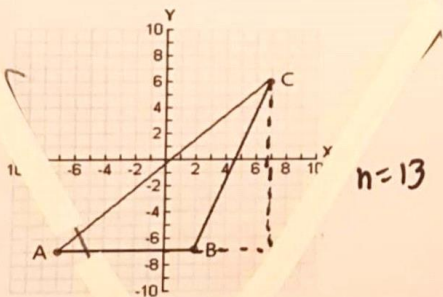Midpoint of DB: $(\frac{-1+3}{2}, \frac{-2+2}{2})$ $(\frac{2}{2}, \frac{0}{2})$

$(1, 0)$ ✓

21. Prove that quadrilateral ABCD shown below is a rectangle by showing the diagonals are congruent.

$AC = \sqrt{(-2-3)^2 + (3--2)^2}$
$= \sqrt{(-5)^2 + (5)^2}$
$= \sqrt{(25) + (25)} = \sqrt{50}$ ✓

$BD = \sqrt{(-3-4)^2 + (0-1)^2}$
$= \sqrt{(-7)^2 + (-1)^2}$
$= \sqrt{(49) + (1)} = \sqrt{50}$ ✓

22. Find the area and perimeter of the following triangle. Simplest form required. Reminder: Draw altitude to find height.

$A(-7,-7)$
$B(2,-7)$
$C(7,6)$

$h = 13$

$AC = \sqrt{(7-7)^2 + (-7-6)^2}$
$= \sqrt{(-14)^2 + (-13)^2}$
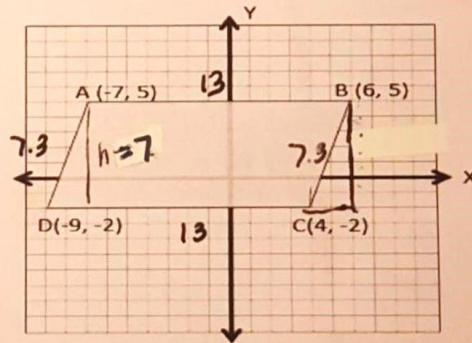$= \sqrt{(196 + (169)} = \sqrt{365} \approx 19.1$

$AB = 9$ unit

$BC = \sqrt{(2-7)^2 + (-7-6)^2}$
$= \sqrt{(-5)^2 + (-13)^2}$
$= \sqrt{(25) + (169)} = \sqrt{194} \approx 13.9$

Area $\triangle A = \frac{1}{2}bh) = (1/2)(9)(13) = 58.5$ units^2

Perimeter (Add all sides) = _____

$9 + 19.1 + 13.9 \approx 42$ or

$9 + \sqrt{365} + \sqrt{194}$ units

23. Find the area and perimeter of the following parallelogram. Simplest form required. Reminder: Draw altitude to find height.

$AB = 13$ units

$BC = \sqrt{(6-4)^2 + (5-2)^2}$
$= \sqrt{(2)^2 + (7)^2}$
$= \sqrt{(4) + (49)} = \sqrt{53} \approx 7.3$

Area: $(A = bh) = (7)(13) = 91$

Perimeter (Add all sides) =

$\sqrt{53} + \sqrt{53} + 13 + 13 = 26 + 2\sqrt{53}$
≈ 40.6

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