

Trigonometric Values Given A Point

$$x^2 + y^2 = r^2 \rightarrow r = \sqrt{(-)^2 + (-)^2}$$

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x} \quad \csc \theta = \frac{r}{y} \quad \sec \theta = \frac{r}{x} \quad \cot \theta = \frac{x}{y}$$

Find the exact values (no decimals) of the six trigonometric functions of an angle θ in standard position whose terminal side contains the given point.

x y

1. (4, -3)

$$r = \sqrt{(4)^2 + (-3)^2}$$

$$r = 5$$

$$\begin{aligned}\sin \theta &= \frac{-3}{5} & \csc \theta &= \frac{5}{-3} \\ \cos \theta &= \frac{4}{5} & \sec \theta &= \frac{5}{4} \\ \tan \theta &= \frac{-3}{4} & \cot \theta &= \frac{4}{-3}\end{aligned}$$

x y

3. (-5, -7)

$$r = \sqrt{(-5)^2 + (-7)^2}$$

$$r = \sqrt{74}$$

$$\begin{aligned}\sin \theta &= \frac{-7\sqrt{74}}{74} & \csc \theta &= \frac{\sqrt{74}}{-7} \\ \cos \theta &= \frac{-5\sqrt{74}}{74} & \sec \theta &= \frac{\sqrt{74}}{-5} \\ \tan \theta &= \frac{7}{-5} = \frac{7}{5} & \cot \theta &= \frac{5}{7}\end{aligned}$$

x y

5. (15, 8)

$$r = \sqrt{(15)^2 + (8)^2}$$

$$r = 17$$

$$\begin{aligned}\sin \theta &= \frac{8}{17} & \csc \theta &= \frac{17}{8} \\ \cos \theta &= \frac{15}{17} & \sec \theta &= \frac{17}{15} \\ \tan \theta &= \frac{8}{15} & \cot \theta &= \frac{15}{8}\end{aligned}$$

x y

2. (-12, 5)

$$r = \sqrt{(-12)^2 + (5)^2}$$

$$r = 13$$

$$\begin{aligned}\sin \theta &= \frac{5}{13} & \csc \theta &= \frac{13}{5} \\ \cos \theta &= \frac{-12}{13} & \sec \theta &= \frac{13}{-12} \\ \tan \theta &= \frac{5}{-12} & \cot \theta &= \frac{-12}{5}\end{aligned}$$

x y

4. (2, 3)

$$r = \sqrt{(2)^2 + (3)^2}$$

$$r = \sqrt{13}$$

$$\begin{aligned}\sin \theta &= \frac{3\sqrt{13}}{13} & \csc \theta &= \frac{\sqrt{13}}{3} \\ \cos \theta &= \frac{2\sqrt{13}}{13} & \sec \theta &= \frac{13}{2} \\ \tan \theta &= \frac{3}{2} & \cot \theta &= \frac{2}{3}\end{aligned}$$

x y

6. (5, 2)

$$r = \sqrt{(5)^2 + (2)^2}$$

$$r = \sqrt{29}$$

$$\begin{aligned}\sin \theta &= \frac{2\sqrt{29}}{29} & \csc \theta &= \frac{\sqrt{29}}{2} \\ \cos \theta &= \frac{5\sqrt{29}}{29} & \sec \theta &= \frac{\sqrt{29}}{5} \\ \tan \theta &= \frac{2}{5} & \cot \theta &= \frac{5}{2}\end{aligned}$$

Key

7. (-1, -7)

$$r = \sqrt{(-1)^2 + (-7)^2}$$

$$r = \sqrt{50} = 5\sqrt{2}$$

$$\sin \theta = \frac{-7\sqrt{2}}{10} \quad \csc \theta = \frac{-5\sqrt{2}}{7}$$

$$\cos \theta = \frac{-\sqrt{2}}{10} \quad \sec \theta = \frac{-5\sqrt{2}}{7}$$

$$\tan \theta = \frac{-7}{-1} = 7 \quad \cot \theta = \frac{-1}{-7} = \frac{1}{7}$$

8. (-1, 1)

$$r = \sqrt{(-1)^2 + (1)^2}$$

$$r = \sqrt{2}$$

$$\sin \theta = \frac{\sqrt{2}}{2} \quad \csc \theta = \frac{\sqrt{2}}{1}$$

$$\cos \theta = \frac{-\sqrt{2}}{2} \quad \sec \theta = \frac{-\sqrt{2}}{1}$$

$$\tan \theta = \frac{\sqrt{2}}{\sqrt{2}} = -1 \quad \cot \theta = \frac{-\sqrt{2}}{\sqrt{2}} = -1$$

9. (-8, 6)

$$r = \sqrt{(-8)^2 + (6)^2}$$

$$r = 10$$

$$\sin \theta = \frac{6/10 = \frac{3}{5}}{10} \quad \csc \theta = \frac{10/6 = 5/3}{6}$$

$$\cos \theta = \frac{-8/10 = -\frac{4}{5}}{10} \quad \sec \theta = \frac{10/-8 = -5/4}{-8}$$

$$\tan \theta = \frac{6/8 = -\frac{3}{4}}{8} \quad \cot \theta = \frac{-8/6 = -\frac{4}{3}}{6}$$

10. (-9, 40)

$$r = \sqrt{(-9)^2 + (40)^2}$$

$$r = 41$$

$$\sin \theta = \frac{40/41}{41} \quad \csc \theta = \frac{41/40}{40}$$

$$\cos \theta = \frac{-9/41}{41} \quad \sec \theta = \frac{41/-9}{-9}$$

$$\tan \theta = \frac{40/-9}{-9} \quad \cot \theta = \frac{-9/40}{40}$$

11. (7, -24)

$$r = \sqrt{(7)^2 + (-24)^2}$$

$$r = 25$$

$$\sin \theta = \frac{-24/25}{25} \quad \csc \theta = \frac{25/-24}{-24}$$

$$\cos \theta = \frac{7/25}{25} \quad \sec \theta = \frac{25/7}{7}$$

$$\tan \theta = \frac{-24/7}{7} \quad \cot \theta = \frac{7/-24}{-24}$$

12. (-2, -2)

$$r = \sqrt{(-2)^2 + (-2)^2}$$

$$r = 2\sqrt{2} \text{ or } \sqrt{8}$$

$$\sin \theta = \frac{-\sqrt{2}/2}{2\sqrt{2}} \quad \csc \theta = \frac{-\sqrt{2}}{2}$$

$$\cos \theta = \frac{-\sqrt{2}/2}{2\sqrt{2}} \quad \sec \theta = \frac{-\sqrt{2}}{2}$$

$$\tan \theta = \frac{-\sqrt{2}/\sqrt{2}}{1} \quad \cot \theta = \frac{-\sqrt{2}/\sqrt{2}}{1} = -1$$

13. (12, 9)

$$r = \sqrt{(12)^2 + (9)^2}$$

$$r = 15$$

$$\sin \theta = \frac{9/15 = 3/5}{15} \quad \csc \theta = \frac{15/9 = 5/3}{9}$$

$$\cos \theta = \frac{12/15 = 4/5}{15} \quad \sec \theta = \frac{12/15 = 4/5}{4/5} = 5/4$$

$$\tan \theta = \frac{9/12 = 3/4}{4/5} \quad \cot \theta = \frac{12/9 = 4/3}{3/4} = 4/3$$

14. (-30, 16)

$$r = \sqrt{(-30)^2 + (16)^2}$$
~~$$r = \sqrt{34}$$~~

$$r = 34$$

$$\sin \theta = \frac{16/34 = 8/17}{34} \quad \csc \theta = \frac{34/16 = 17/8}{16}$$

$$\cos \theta = \frac{-30/34 = -15/17}{34} \quad \sec \theta = \frac{34/-30 = 17/-15}{-15}$$

$$\tan \theta = \frac{16/-30 = -8/15}{-15} \quad \cot \theta = \frac{-30/16 = 15/8}{15}$$